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ON THE QUESTION OF DAY-TO-DAY FLUCTUATIONS IN THE DERIVED VALUES OF THE SOLAR CONSTANT

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[Washington, September 2, 1925]

SYNOPSIS

Frequent use is made of the short term "solar constant" to denote the derived values of the intensity, expressed in appropriate thermal units, of solar radiation as if measured just outside the atmosphere of the earth when at its mean distance from the sun.

Determinations of the solar constant show small but important fluctuations from day to day. This investigation is a search for evidence as to what part, if any, of these and of other short-period fluctuations, should be ascribed to solar changes, and what part, if not all, must be assigned to the inevitable errors of derivation.

Unusual methods of analysis are required, because the total variation due to all causes is so small that it is entirely plausible that all of it may be nothing but errors of measurement. At the same time it is possible some solar variation may exist.

SECTION I. The mathematical equations for computing the solar constant are given with extensions drawn from statistical theorems for impersonally measuring, comparing, and correlating variations in observational data.

SEC. II. Securing highly accurate values of the solar constant in absolute magnitude is a difficult problem by itself and is wholly foreign to this study.

Evidence for and against day-to-day and other variations in solar intensity can be secured from pyrheliometer readings alone.

How this is done is shown by a sample analysis of the latest and best observations thus far published in full, namely, for Calama, Chile, July, 1918, to July, 1919. The pyrheliometer is the basic instrument for all solar constant measurements. Its errors are smallest, most certainly known and most constant, and when properly standardized it is the most comparable of all the instruments employed.

It is not difficult to show that nothing but the sun and errors of derivation can cause variations of the solar constant at a single station. If half the total variation found at Calama is assumed to be due to the sun, analysis shows the probable departure of any daily value from the mean for the year due to the sun to be ± 0.0083 calorie. To ascribe this small possible variation to the sun is to assume that the total variation at the Calama station due to all causes is the irreducible minimum of total variation to be found when values are available from many equally good stations.

SEC. III. A graphical tabulation is given to aid in the interpretation of correlations between pyrheliometer and other observations.

SEC. IV. Consecutive values of solar constant values from 1902 to 1924 are charted and probable variations evaluated and illustrated.

SEC. V. Any annual periodicity in solar constant values is prima facie evidence of terrestrial influence. Small but important summer and winter effects of this kind based on fully 3,000 daily values are shown in a striking manner, including the physically inconsistent values for the station at Harqua Hala, which are found to be correlated with the values at Montezuma in an artificial way.

SEC. VI. An example is given of how solar variations can be segregated from variations due to errors, when simultaneous observations are available from one or more pairs of independent stations by the solution of three simultaneous equations between solar variation and the two other unknown variations caused by station errors. Incidentally, it is shown how to ascertain whether the three unknowns are interdependent and thus how to interpret in a rational way the results secured.

INTRODUCTION

For a period of more than 20 years the Astrophysical Observatory of the Smithsonian Institution has been securing observations of the thermal intensity of the sun's radiation as if measured just outside the atmosphere of the earth when at mean solar distance. The derivation of such results from observations made at the bottom of our atmosphere, even when stations are located on mountain tops or high plateaus, is beset with serious and uncontrollable errors due to the clearing up or hazing up of the local atmosphere resulting from the everchanging states of dust, water content, and turbidity of the air column through which the incoming radiation must pass before reaching the measuring instruments.

The frequency of such observations and their order of accuracy have steadily increased. Values of the solar constant are now being secured by the Astrophysical Observatory from one or both of two stations in opposite hemispheres, one at Montezuma, Chile, about latitude 22° 28′ S., longitude 68° 56′ W., altitude nearly 10,000 feet; the other at Harqua Hala, Ariz., latitude 33° 45′ N., longitude 115° 15′ W.; altitude 5,680 feet. It is reported that plans are in hand to establish a third station at some suitable place in the Eastern Hemisphere.

The best observations by the so-called long or Langley bolographic method show a probable variation due to all causes of less than 1 per cent of the total intensity. Values obtained by a short method using an empirical instrument known as the pyranometer show a probable variation of less than one-half of 1 per cent.

In the making of some kinds of measurements we know with certainty beforehand that the quantity we measure remains indefinitely constant within the limits of precision of our measures. We then properly attribute all observed variations to errors of determination. In many other kinds of measurements, well illustrated by determinations of the solar constant, we are in doubt. Some of the observed small changes may be due to something besides errors of determination, as in this case to the sun itself. If such changes were large as compared with the unknown errors, little or no uncertainty would arise. On the other hand, if the possible solar changes are so small as to be incapable of direct measurement and individual identification, then very unusual statistical methods must be invoked to get at the facts, that is, to disentangle the small hidden solar variations from the total due to all causes.

As soon as good values of the solar constant were obtained, the claim for important day-to-day changes in solar intensity began to be made. No one, however, so far as I am aware, has made the unusual kind of analysis required to prove that such day-to-day fluctuation was partly of solar origin. The problem is necessarily a difficult and more or less inconclusive one.

Ten years ago, when the claims for solar variation began to be confidently asserted, the statistically measured total variation of high-grade observations was about 1.2 per cent. If a skeptic of that day had conceded that *half of this statistically measured total* might be ascribed to the sun he would to-day be facing the unpleasant reductio ad absurdum that the *part* formerly conceded is now greater than the *whole*, so greatly has this whole been reduced in the latest high grade observations.

Notwithstanding all such representations as the foregoing, Doctor Abbot is convinced that his observations show important systematic changes of solar intensity from day to day, week to week, etc. He is further convinced that studies by Mr. Clayton show important correlations between the values of the solar constant and the weather at various places on the surface of the earth, of such a nature that despite all their errors, the changing values of the solar constant become a trustworthy basis for short and long range forecasts of the weather.[1]

The interest of the Weather Bureau in these investigations has constantly been very great, and it has examined with special care the values of the solar constant as these have been published at intervals by the Astrophysical Observatory of the Smithsonian Institution. Its studies are necessarily based only upon the final values as published, which are but a fragment of the mass of original observations, and in these published data the Bureau fails to find a sufficient basis for the view that short-period solar fluctuations do exist. Even conceding the possibility that a *part* of the present small total variation due to all causes may be ascribed to the sun, it must still be shown that this part exists, and, small as it certainly is, that it is physically sufficient to cause direct daily effects upon the weather of such a magnitude that they have a value in weather forecasting.

These considerations show how necessary it is that the analysis be made without further delay.

There are, no doubt, astronomers and meteorologists who may, without themselves undertaking a critical analysis of the observational data, tacitly accept at their full face value the published claims for important solar variability from day to day and who may wonder as to the forecasting possibilities of such a characteristic of solar activity.

It would be quite unwarranted to say that the thermal radiation of the sun is unchangeable from day to day, or from season to season. In the face of all that has been revealed as to the sun's physical features and activity in the way of spots, flocculi, faculae, coronal streamers, prominences, etc., there are abundant physical grounds for suspecting that changes of intensity of radiation are occurring all the time. Accordingly, if accurate observations of the solar constant had never been made, claims of day-to-day and other variations could not be questioned or refuted. Fortunately, however, a very large body of highly accurate values of the solar constant are accessible in the publications of the Astrophysical Observatory.

For the information of many who doubtless would like to know quantitatively the significance of such day-to-day fluctuations as may exist, it is the purpose of this study to examine the mute testimony of the published data, as nearly as the nature of these data permits.

Observations[2] of varying exactitude have been secured since the early efforts during the years 1902 to 1907 to develop apparatus and methods, at Washington, D. C. Here the sky conditions were usually highly unfavorable, and naturally the variations of derived values of the solar constant were rather great. Serious observational work began, however, in 1905, when a station equipped with pyrheliometer and bolograph was established at Mount Wilson, Calif.

The observations are not, of course, all of equal excellence. Doctor Abbot has characterized them[3] as follows:

> Really, to speak in a figure, the Washington data of 1902 to 1907 were Prehistoric. As for Mount Wilson results of 1905 to 1908, inclusive, before the invention of the silver disk pyrheliometer, or Fowle's method for estimating total atmospheric humidity, and while we yet used a flint glass prism limiting our spectrum at the H and K lines in the violet—this work is Ancient. Excluding altogether July and August, 1912, the year of the eruption of the Katmai volcano, all Mount Wilson work of 1909 to 1920 can be classed as Medieval. We had then but one station, operating only in summer. We obtained only one determination per day, subject to error from changes of sky transparency and also to errors of computing in the enormous multiplicity of computations used in the reductions of results by Langley's fundamental method. The period from January, 1919, to the present is of another order of accuracy, and represents the Modern period.
>
> All of the Mount Wilson work, excluding altogether July and August, 1912, is useful in the form of averages. It is only since January, 1919, when we have had several determinations each day by a method [pyranometer,-*C.F.M.*] which avoids errors from the variability of the sky, and much of the time have received results from two stations, that individual values have begun to deserve some confidence.

Notwithstanding this severe disparagement of the bolographic and older work, we have long marveled at the general high order of accuracy secured by Doctor Abbot, not excepting even those observations which were influenced by Katmai dust during 1912 and 1913. Here, fortunately, we have positive evidence, which only a violent volcanic eruption could produce, of the extent to which atmospheric influence on incoming radiation can cause spurious variations of the solar constant.

The systematic observations with the pyranometer fall in a class by themselves and have a probable variation of less than half that of values secured by the bolograph. Nevertheless, the pyranometer is entirely an empirical instrument and its absolute accuracy can not be as great in the long run as that of the bolograph, from which all the empirical coefficients of the pyranometer must be derived.

Mr. Clayton makes extensive use of the Mount Wilson bolographic observations for the years 1913, 1915, and 1918, often in small groups of extreme values only, to establish his correlations of supposed changes of solar constant with weather changes. Accordingly, we also shall use these data, but only in the form of monthly and annual averages, the accuracy of which is much greater than that of single or even of several daily values, especially when the latter, chosen because they are extreme, are therefore most likely to be affected by error larger than the mean error.

Our analysis must speak for itself as to its sufficiency and soundness, but we are glad to emphasize that the general high excellence of the observational data not only justifies but invites critical statistical examination, and rewards the effort by gratifying consistency and definiteness in the results obtained.

There is no pretense in this paper to an exhaustive analysis of all the pros and cons of variations in the sun's thermal radiations. We confine our analysis to a single problem:

In the derived day-to-day values of the solar constant are found greater or lesser irregular changes. What part of these, if any, is due to changes in solar intensity, and what part to wholly unavoidable atmospheric influences and other errors of measurement?

[1] Report on the Astrophysical Observatory, 1924, Appendix 7. Abbot, C. G., Solar variation and forecasting. Smiths. Misc. Coll., vol. 77, no. 3, 1925.

[2] Abbot, C. G., and colleagues. Annals of the Astrophysical Observatory of the Smithsonian Institution, Vol. III, 1913, Vol. IV, 1922, and Provisional Values of the Solar Constant, August, 1920, to November, 1924, Smithsonian Miscellaneous Collections, vol. 77, no. 3.

[3] Abbot, C. G., Solar Variation and Forecasting. Smiths. Misc. Coll., vol. 77, no. 5, pp. 2-3.

Fully alert to the great meteorological importance of consequential changes in the solar constant over both short and long periods of time, *we regard it as of paramount importance to seek out a quantitative answer to the question proposed. It is futile to hope to establish any scientific basis for weather forecasting on supposed changes of solar constant before we know that the constant does change from day to day, and if it does, how much.*

The discussion in Chapter V of Volume IV of Annals of the Astrophysical Observatory on methods for evaluating errors, is unsatisfactory and misleading because, pointing out the numerous ways in which errors can occur, it assigns to some of them very approximate values, and even these are based on special and individual cases. The only acceptable measure of errors affecting observations for, say, a whole year, under all kinds of atmospheric conditions, is some such measure of fluctuation as the standard deviation. This definite statistical index of scatter of the derived values measures all the variations. Adequate statistical proof must support any claim that part of them are of solar origin.

This preliminary analysis necessarily must deal with the short-period solar changes, leaving the long-period, slow, progressive changes to be dealt with in later studies.

The subject will be discussed under the following captions:

I. Theoretical considerations.
II. Analysis of pyrheliometer readings at Calama, Chile, using standard deviations.
III. Analysis of Calama data by correlations.
IV. General examination of the variability of all values of the solar constant by bolograph and pyranometer.
V. The 12-month period in solar constant values for northern and southern hemispheres.
VI. Solar variations computed from observations at independent stations.
VII. Conclusion.

I. THEORETICAL CONSIDERATIONS

SYMBOLS AND NOMENCLATURE

a_λ. Coefficient of transmission of the air for monochromatic thermal radiation of the sun of wave length λ.

a. Apparent coefficient of transmission for polychromatic radiation as measured by pyrheliometers and black body absorbers of total radiation.

I_1, I_2, I_3. Intensities outside the atmosphere of various spectral beams of monochromatic radiation.

I_0. The true errorless intensity of total solar radiation outside the atmosphere $= I_1 + I_2 + I_3 + \ldots$, the true solar constant.

m. Relative air mass at the same station as dependent upon the sun's zenith distance.

z. Angular distance of sun from zenith at time of an observation.

$p. w.$ Atmospheric moisture measured as precipitable water.

A_1, A_2 .... A_m. Intensities of total radiation at a station as measured by the pyrheliometer or like instrument at different air masses *1*, *2* .... *m*

i_1, i_2, i_3 intensities of the radiations I_1, I_2, I_3, etc., after transmission through air mass m.

h_1, h_2 .... h_3. The height in millimeters or other *linear* unit of the ordinates on the bolographic trace or energy spectrum curve as observed. Such ordinates are deemed sufficient to show the *relative* thermal intensities in the solar spectrum, and by summation, after the application of a complicated series of corrections, are regarded as directly proportional to simultaneously observed values of the pyrheliometer A_m.

E_0. The Smithsonian Institution's symbol representing the *solar constant*, including all its errors of terrestrial origin.

A_0. A convenient and helpful analytical quantity which satisfies a certain equation given later. In familiar language, it is a hybrid solar constant found by the extrapolation of logarithms of pyrheliometer observations at different air masses to zero air mass by the straight line of best fit. The significance of the quantity is purely analytical, not physical.

A_s, A_q. Values for the solar constant obtained as explained in Table 2. Their significance is purely analytical, not physical.

NOTE.—Each of the foregoing quantities representing thermal intensities is subject to a variation depending upon the earth's distance from the sun at the time. All such variations are assumed to be completely excluded from data before analysis, by reduction to the earth's mean solar distance.

The following symbols relate to fluctuations and their statistical measurements:

σ, Standard deviation, occasionally called scatter and based on departures from the mean.

t, x, y, i, as subscripts, signify the causes of the fluctuations, as total causes, errors at stations X or Y, and due to sun, respectively.

V, the variate difference, the difference between consecutive values.

ΔI_0 represents the variate difference between real solar intensities.

In $\overline{E}_0$, $\overline{V}$, $\overline{V^2}$, $\overline{\Delta I^2}$, the superior bar indicates that these are mean values.

v, a departure from a mean value, a residual.

md the mean deviation or the average sum of departures from the mean without regard to sign.

According to the well-known Bouguer-Langley exponential law of atmospheric transmission of radiation, a single beam of monochromatic radiation of original intensity I_1 will have an intensity, i_1, at the bottom of an air mass m, given by the equation

$$i_1 = I_1 a_1^m \qquad (1)$$

in which a_1 is the coefficient of transmission for radiation of the particular wave length of the beam in question.

Now, solar radiation is polychromatic; whence, there are many beams of varying wave lengths and varying intensities I_1, I_2, I_3, ... etc., each with its appropriate coefficient of transmission a_1 a_2 a_3, etc. After transmission through air mass m these have the several intensities—

$$I_1\, a_1^m,\ I_2\, a_2^m,\ I_3\, a_3^m\ \ \ldots,\ \text{etc.}$$

When properly standardized, the pyrheliometer measures the total thermal radiation transmitted to its place of exposure at the bottom of the ocean of atmosphere. If this total for a given air mass m is A_m, then

$$A_m = I_1\, a_1^m + I_2\, a_2^m + I_3\, a_3^m + \ldots \qquad (2)$$

and if the original intensity of the total radiation is I_0 then

$$I_0 = I_1 + I_2 + I_3 + \ldots \qquad (3)$$

Equation (1) is widely accepted as rigorously exact and accordingly constitutes a satisfactory analytical basis for the present effort to evaluate from the actual observations day-to-day and other frequent short-interval variations of solar radiation. Any such variations must express themselves, first, as variations in I_1, I_2, ... etc., either separately or collectively. If they do not at once neutralize and nullify each other, all such variations, however they may occur, must reflect themselves as variations in I_0 and A_m or other observations of radiant intensity that we may be able to make. Equation (1) establishes definite functional and extremely simple relations between the one dependent variable A, (dropping the subscript m) and the three wholly independent variables m, $I_0=\Sigma I_\lambda$ and Σa_λ. It is fortunate that apart from the irregular variations we call errors, equation (1) seems to include every known cause of variation affecting the quantities involved. Moreover, as will appear later, when the quantities are in logarithmic form the relations become strictly linear and therefore greatly facilitate correlation and render the interpretation of correlation coefficients the more definite.

Of the three independent variables, m is the only one that is under even partial human control. If we can observe the pyrheliometer at a particular station with the sun exactly in the zenith, then we may assign to m the value 1. If the sun is at an angle z from the zenith, then $m=\sec z$ (approximately). We still remain ignorant of any mathematical function by which to equate air masses at one station with those at far distant stations.

The variables I_1, I_2 .... a_1, a_2 ...., etc. in equation (2) are independent. The sun alone controls the values of I_1, I_2, I_3, ..., etc. On the other hand the coefficient of air transparencies a_1, a_2, a_3 represent constantly changing conditions of the earth's atmosphere, not directly ascribable to the sun and wholly beyond any human control except such as may be expressed by a choice as to where and when we make pyrheliometric readings.

It is plain that there is no rigorous and at the same time workable transformation between (2) and (3) by which I_0 can be equated directly to A_m and the other variables. However, it has long been known that equation (1) for monochromatic radiation can be used also with polychromatic radiation, either by disregarding a small outstanding variable, p, due to polychromatic radiation, or, as we prefer to do, by writing p into the equations for ultimate evaluation. This course is especially appropriate in analyzing observations from stations at various high altitudes overlain by the driest, most transparent air masses possible to attain. In such cases $a_1^m\ a_2^m\ a_3^m$, etc., are most nearly unity and A_m approaches I_0; that is, p, seemingly best expressed as a ratio, is then nearly constant and nearly equal to 1.

These considerations give, after dropping the subscript m,

$$A=A_0\,a^m \qquad (4)$$

$$A_0p=I_0=\frac{i_1}{a_1^m}+\frac{i_2}{a_2^m}+\frac{i_3}{a_3^m}+\cdots \qquad (5)$$

In (4) a is now the apparent transmission coefficient for the whole polychromatic beam of radiation measured by the pyrheliometer. In (5) $p=\frac{I_0}{A_0}$ is the new variable ratio permitting A_0 to be equated to I_0, which here represents not an observation, but the true solar constant as an independent variable. A_0 in (4) and (5) has no real physical significance and is simply the value of A in (4) when $m=0$.

The quantity p can not be evaluated from pyrheliometric observations at a single station, but we are now concerned only with variations in A_0 and p and it will suffice to replace p by a constant value p_0 and a variable part which for all practical purposes can be classed as part of the unavoidable errors of observations, as will appear later.

Writing equation (4) in logarithmic form we get

$$\log A=\log A_0+m\log a \qquad (6)$$

and by the familiar straight line extrapolation of low and high sun observations of A to zero air mass we get values of A_0 and a.

Pyrheliometry.—Up to this point all equations are as rigorously exact as the Bouguer-Langley law of atmospheric transmission permits. Each term is regarded as an errorless value or fact. Now, however, we must pass to fallible human observations of the pyrheliometer and other inexact measurements of A_0, m, and a. Indeed, in evaluating A_0 and a in (6) by low and high sun observations, we make an assumption which both reason and experience tells us can rarely or never be satisfied, namely, that *both I_0 and a remain constant during the several hours required to make the necessary low and high sun observations of A.* If the *transparency of the air* and the *solar intensity* change irregularly *from day to day*, how futile it is to assume, as we are prone to do, that these variables obligingly remain constant during several hours each day while we make observations of intensity at different air masses. Every failure of the assumption to be satisfied is necessarily and faithfully extrapolated to zero air mass as an error, and there it appears to be a fluctuation of the solar constant whether the result is A_0 or E_0, because of course the bolograph is powerless to exclude errors due to a fallacious assumption about the constancy of the atmosphere or of the sun.

Few observations made anywhere in the world are quite free from evidences of this insidious cause of error, of which at least a part must be ascribed to the sun if we insist upon appreciable day-to-day changes of solar intensity.

High grade pyrheliometer readings clearly show that measurements of total polychromatic radiation at different air masses require a *curved* line to properly represent their trend and that for equation (6) we should write

$$y=y_0+b\,m+c\,m^2 \qquad (7)$$

in which the logarithms are represented by the simple letters y, y_0, b and c. A part of this curvature can be ascribed to the complete extinction of some radiation at times of low-sun observations. While the effect of such losses is small, the question deserves more careful examination than it seems to have received thus far.

Analytical relations can be shown justifying the use of a power series to represent polychromatic radiation and equation (7) introduces the first term of such a series. A very few trials show that such a quadratic equation fits group mean values in a highly satisfactory way. The equation, however, is all but worthless for the extrapolation of *daily* values, because the large variations in such observations caused by the failure of a or I_0 or both to remain constant gives entirely spurious values to b and c. Moreover, the intercept y_0 at air mass zero has no physical meaning. Quantitatively, it is like A_0 in equation (4). It is simply the value of y in equation (7) when $m=0$.

Notwithstanding these limitations, equation (7) promises to be highly useful in the analysis of large group values of data for the study of long-interval changes in I_0.

Bolometry.—The series of terms i_1, i_2, i_3, etc., in equation (5) represents the intensities of the numerous beams of monochromatic radiation as they reach the bolograph and there represent the energy of the solar spectrum on the bolographic trace. It is impossible to measure these intensities except in a purely relative way, giving rise to a new series of quantities which are mere linear measures of bolographic ordinates h_1, h_2, h_3, etc.

The process by which these ordinates can be transformed into thermal intensities, extrapolated to zero air mass and finally converted into the thermal magnitude E_o is highly tedious, complex, and entails numerous corrections for errors, losses, etc. It is fully described in the various Annals of the Astrophysical Observatory. Since the invention of the pyranometer or "short method" of observing, Doctor Abbot has practically discarded the bolographic or "long method" for securing daily values of E_o (but not for determining "function transmission curves"[4]) because errors of individual determination are so serious that reliance can be placed only upon group means of values thus found.

While the bolographic method is the only fundamental one for getting values of the solar constant at a single station, its errors nevertheless probably exceed 1 per cent or more if we fairly include the long train of secular and systematic errors of a semiconstant character. The only way to learn something definite about such errors is to maintain two or more *completely independent* sets of instruments in operation side by side for a whole year or more. This would permit simultaneous measurements at the duplicate stations, of the *same thermal energy* transmitted through the *same air mass*, and all differences in daily values could then be due to nothing but instrumental or within-the-observatory errors.

Pyranometry.—The pyranometer is an instrument which measures the brightness of the sky in a limited annular area around the sun. The process by which it is possible to get values of the solar constant from its use is entirely empirical and arbitrary. The method is described in Volume IV of the Annals of the Astrophysical Observatory, and its use requires both pyrheliometer and bolographic records as a basis. Sometimes several values of the solar constant can be secured in the same half day, giving a mean value of seemingly small error.

Whichever method of observing is followed, unavoidable variations in day-to-day values are necessarily present, due solely to errors. Wherefore, in order to preserve the analytical integrity of our final equations, and especially to recognize the highly important part which daily, weekly, and seasonal atmospheric states play in causing entirely fictitious variations in the solar constant, we shall introduce the total errors, X, Y, Z, due to all causes in the equation representing a final single value.

$$I_o = \begin{cases} (A_o - X)\ p_o & \text{Pyrheliometer} \\ E_o - Y & \text{Bolograph} \\ E_{wm} - Z & \text{Pyranometer} \end{cases} \qquad (8)$$

These are now the final rigorous relations between I_o, as the true solar intensity treated as a wholly independent variable, and the faulty measurements we may make of it by either or all of the instruments named. The equations are practically self-evident and axiomatic, but the detailed relations of the terms to direct observations have been set out in the preceding equations (1) to (7).

Gaussian distribution.—When we contemplate and talk about day-to-day solar variations our language is vague and indefinite until we indicate the nature of the *frequency distribution* of such variations. Direct observational evidence on this point is wanting; the total variation of the best *derived values* always conforms quite closely to the Gaussian distribution. The better, the more numerous, and the more homogeneous are the data, the closer is the conformity. Now, since the distribution of accidental errors is always approximately Gaussian, we have no choice from present evidence but to assume that *solar variations* also are Gaussian; otherwise, either there are no solar variations at all, or they are such a small part of the total variation, as to make no impression on the normal distribution which represents total errors.

Solar variations v. errors.—It is quite possible that changes of solar intensity as measured at the earth may occur every time a sun spot passes nearly centrally across the sun's disc, and some large temporary and infrequent changes may be caused in this way. However, these are a class of effects by themselves and must be so studied. On the other hand, years of observation show that over long periods day-to-day fluctuations, the relation of which to the sun is at least extremely doubtful, occur constantly, and that variations due to all possible causes have become smaller and smaller as the errors of measurement have been reduced. The fluctuations due to errors must always have a finite value. Hence the improvement in observational methods has now confined within very narrow limits the range of possible solar changes, which was formerly considerable.

It is, of course, wholly impossible to get daily values of X, Y, or Z, but we can always make numerous comparable observations and compute from these the standard deviation or other index of scatter due to all causes. Provided solar changes and errors are uncorrelated (i. e., σ_i not large or small according as σ_x is large or small), we can, from the well-known rules of least squares, write the equation expressing the relation of this *total variation* to variations caused by errors and by the sun as follows:

$$\sigma_t = \sqrt{\sigma_i^2 + \sigma_x^2}, \qquad (9)$$

in which the standard deviations σ are designated by subscripts which signify: t, the total variation due to all causes; i, the variation due to solar variability; and x, that due to errors of all kinds.

As long as day-to-day solar changes are no greater than the best modern observations show to be possible, our only source of real information about solar changes is to be found through some bona fide solution of equation (9). If solar changes are zero then the total changes are simply the total errors. And since the errors can never be zero, solar changes are only a part of the total changes.

The problem of two or more stations.—Simultaneous observations at two or more stations are so scanty that apparently no one has attempted any considerable analysis of such data. It seems well, however, to write out the statistical equations which can be employed. The data at each station furnish an equation of the type of (9), thus

$$\sigma_t = T_x = \sqrt{\sigma_i^2 + \sigma_x^2} \text{ for station } X \qquad (10)$$

$$\sigma_t = T_y = \sqrt{\sigma_i^2 + \sigma_y^2} \text{ for station } Y \qquad (11)$$

The subscripts x and y connote the variations due to errors pertaining to the stations X and Y, respectively. Each equation separately contains two unknowns and is therefore indeterminate. The two equations contain three unknowns and are still indeterminate. However, let us find the difference between simultaneous values

[4] MONTHLY WEATHER REVIEW, February, 1923, p. 74, and April, 1923, p. 188.

at the two stations and evaluate the total variation T_{xy} of such differences. Then we get,

$$T_{xy}=\sqrt{\sigma_x^2+\sigma_y^2} \qquad (12)$$

Thus we secure three simultaneous equations with three unknowns, permitting *absolute* evaluation. However, the result will have little or no physical meaning unless the unknowns are entirely independent. There must be no secular or systematic fluctuations in the simultaneous values due to other causes than the sun. Annual periodicities in solar constant values, and their correlation with air transparency and other terrestrial conditions, will generally serve to vitiate the physical significance of the results drawn from the three simultaneous equations.

The mathematician recognizes, of course, that securing a seemingly rational and finite value of σ_1 in the solution of the three equations for a group of simultaneous observations is no proof of solar variability. Having *assumed* solar variability, a solution of the equations simply apportions to solar variation such part of the total variation as best satisfies the observations at the two stations under the assumed conditions. Some sets of observations may give imaginary roots, and it is obvious that errors of observation can be neither zero nor imaginary.

Solar variation can be shown by these equations only when the results are based on several groups of data from wholly independent stations. As pointed out above, equations of the type of (9) are valid only if σ_1 is unrelated to σ_x or σ_y in magnitude.

Possibilities of the variate difference.—If a, b, c, d - - - - m, n are homogeneous consecutive values of any variant, then $b-a$, $c-b$, $d-c$ - - - $n-m$, and $a-n$ constitute the *complete* sequence of variate differences. This statistical datum seems to be capable of serving many useful purposes. Apparently its use has never been invoked in the critical analysis of solar radiation data.

Emphasis is placed upon taking the complete sequence of differences in-a-ring by adding to the consecutive differences usually taken, the difference between the first value and the last. This is literally a complete integration around a cycle of changes and affords important analytical advantages.

There are important similarities and important differences between the departures from the mean in a body of data and their statistical cousins the variate differences. The algebraic sums of the departures from the mean and variate differences in-a-ring are zero. The average departures from the mean without regard to sign, commonly called the mean deviation *md*, is wholly *independent* of the order of succession of the variant. The average sum of the variate difference disregarding signs is wholly *dependent* upon the order of succession. The natural order of succession may give a value of the mean variation, $\overline{V}$, quite different from a fortuitous order. If the order of succession is fortuitous and the distribution Gaussian, the following important relation holds:[5]

$$\frac{\overline{V}}{md}=\sqrt{2}$$

Just as we measure scatter or variability by the mean square of the departures, so the scatter of day-to-day values of the solar constant can be measured by the quantity $\frac{\Sigma V^2}{n}=\overline{V^2}$, and at a single station we will have

$$\frac{\Sigma V^2}{n}-2\frac{\Sigma X^2}{n}=\frac{\Delta I_o^2}{n}$$

This equation is easily derived from any of the observational equations in (8) by forming the variate difference V and ΔI_o. In simplified nomenclature, using the superior bar to represent mean values we have

$$\overline{V^2}-2\sigma_x^2=\overline{\Delta I_o^2} \qquad (13)$$

Simultaneous observations at a second station, together with an equation based upon the difference between the values at the two stations gives

$$\overline{V}_x^2=\overline{\Delta I_o}^2+2\sigma_x^2 \text{ Station } X \qquad (14)$$

$$\overline{V}_y^2=\overline{\Delta I_o}^2+2\sigma_y^2 \text{ Station } Y \qquad (15)$$

$$\overline{T}_{xy}^2=\sigma_x^2+\sigma_y \qquad (16)$$

in which the mean variations and errors for the respective stations are designated by the subscripts x and y.

Thus we have, by using departures from means in the one case and variate differences in the other, two different means of securing quantitative evaluations of the variations of solar intensity. As soon as two or more really independent stations supply observations of the solar constant as free as possible from annual periodicities and correlations with atmospheric and climatic features, we may hope to gather some worth-while evidence for or against day-to-day and other solar variations.

Observations at different air masses.—Equation (9) can be used in the analysis of observations of actual intensities at different air masses. Day-to-day variations at a single air mass can be ascribed to only three causes:

(1) Errors of observation X—never zero.
(2) Atmospheric depletion a—always changing.
(3) Solar changes—if they exist.

If a and I_o remain constant while air mass changes, the scatter due to depletion of incoming radiation is directly proportional to the air mass, m, and the total variation due to all causes is

$$\sigma_t=\sqrt{\sigma_x^2+\sigma_1^2+m^2\sigma_1^2} \qquad (17)$$

in which σ_1 = the variation at air mass 1 due solely to day-to-day changes in a. It must be understood that in this equation all changes due to failure of a or I_o to remain constant during observations are classed as errors and appear in σ_x. Since σ_1 is wholly independent of m, and since nothing is known as to how σ_x may vary, as it must, with m, equation (17) can be used to evaluate only σ_1^2 and $(\sigma_x^2+\sigma_1^2.)$ and will be applied in this way later.

The analytical and other principles presented in the foregoing appear to be a sound and sufficient guide for the detailed analysis of the various groups of data. This will now be taken up.

II.—ANALYSIS OF PYRHELIOMETER READINGS AT CALAMA, CHILE, USING STANDARD DEVIATIONS[6]

The pyrheliometer is the fundamental and indispensable instrument for all measurements of solar intensities. Its errors are smallest, most certainly known, and most nearly constant of all the instruments employed. When standardized by comparison with an absolute or invariable normal, the pyrheliometer would be entirely sufficient by itself to secure values of the solar constant if the

[5] Marvin, C. F., MONTHLY WEATHER REVIEW, Sept. 1924, 52: 441. Woolard, E. G. MONTHLY WEATHER REVIEW, March 1925, 53: 107.

[6] It is a pleasure to acknowledge the assistance rendered by the several members of the Weather Bureau staff who have so effectively cooperated during the preparation of the analyses presented in this paper.

radiation to be measured were monochromatic. The long train of mirrors, prisms, bolometers, galvanometers, pyranometers, and the elaborate procedure and empirical corrections entailed by their use are necessary solely to overcome the errors which polychromatic radiation introduces when the pyrheliometer alone is employed.

This limitation upon the pyrheliometer applies only to securing the *absolute* value of the solar constant. Day-to-day changes in those values, if they exist at all, *must appear in readings of the pyrheliometer.* The bolometer is simply an empirical analyzer whose function is solely to put the observed total intensities at different air masses in the form of spectral ordinates, h_1, h_2, h_3, etc., so that the Langley-Bouguer equation for extrapolation to zero air mass can be applied thereto. The analysis must not be allowed to add to or take from the total heat registered by the pyrheliometer.

The mass diagram Figure 1 contains a mine of information for the earnest student. Each dot individually is the logarithm, reduced to mean solar distance, of the observed intensity at the particular air mass represented by its abscissa. It is as nearly an errorless observational fact as the art of pyrheliometry, combined with conscientous observing under cloudless skies, permits. The dots falling on or near any vertical line represent observations at the same air mass at intervals of one or more whole days.[7] The variations in intensity such dots show are caused in part by small instrumental errors but chiefly by changes from one day to the next either in solar intensity I_0, or in air transparency a, or to both causes. At the extreme right, under conditions of low sun (air mass 5), the intensities are small, the air mass changes rapidly from minute to minute, and the errors are relatively larger than for high sun, that is for air masses between 1 and 1.5. Here intensities are high, air mass

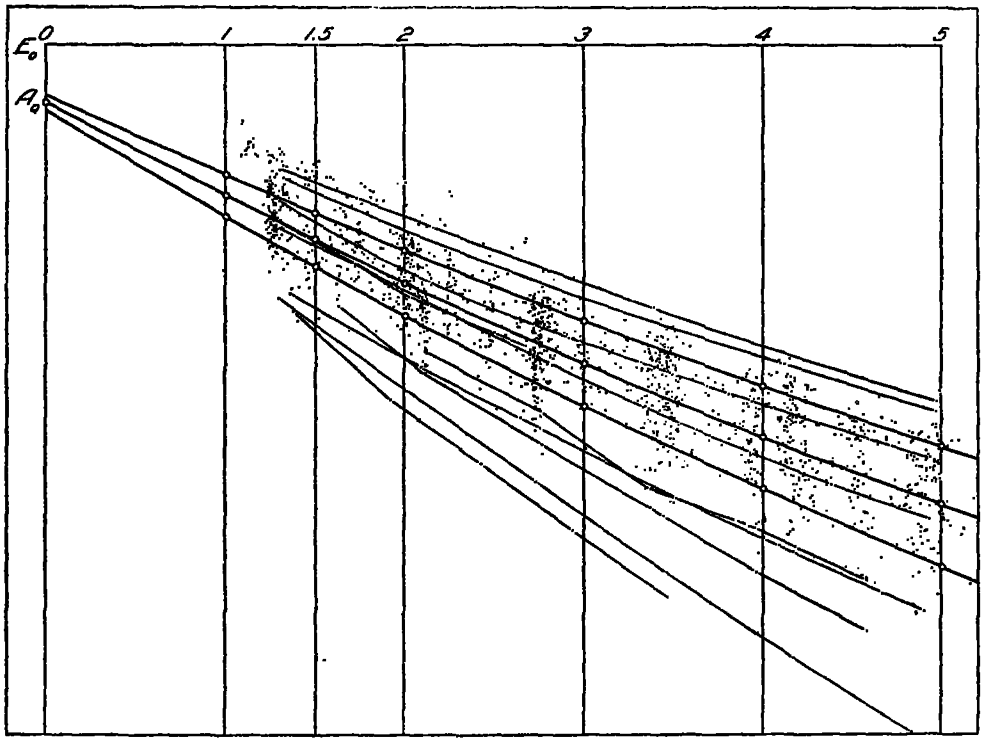


FIG. 1.—Mass diagram of 239 pyrheliometer readings at Calama, Chile. Tables 27 and 28, Annals Astrophysical Observatory, Vol. IV. A few broken lines join values for the same day. The annual mean values at air masses 1 to 5 are shown extrapolated by quadratic equation (7). Departures above and below the mean, $\pm\sigma_m$, for each air mass are similarly extrapolated. A few outlying dots in the upper part of the diagram represent observations at high-level stations other than Calama, and not included in this analysis

In a like manner the pyranometer also is an empirical device. It is used as a highly arbitrary substitute for the rigorous law of extrapolation to zero air mass. Obviously, it also can not be allowed to add to or take from the true amount of atmospheric depletion. Therefore, all *fluctuations* of solar constant shown by either or both of these empirical devices which can not be definitely shown to be already registered in the total heat or parent data secured by the pyrheliometer must at once be suspected as artificial and introduced by the empirical devices.

Recognizing the fundamental and basic nature of the original pyrheliometric observations, their analysis is therefore, our first objective.

The year as a unit of record.—A full year is the natural and only safe climatic interval to employ in the analysis of solar constant values which, experience shows, are permeated through and through with local and terrestrial atmospheric effects.

changes very little in several minutes, and the order of accuracy is generally higher. However, observations at any given air mass frequently show irregular changes of intensity from minute to minute, due either to variations of a or of I_0 or of both. From air mass 5 to air mass 1 the observations, individually nearly errorless, collectively are permeated with variations caused by the failure of a and I_0 to remain constant with the lapse of time between observations.

To analyze these more than 1,400 observations, the logs of intensities were assembled to give individual daily values of intensity at integral air masses 1.5, 2, 3, 4, and 5 for each of the 239 days, making a total of about 1,200 values. These were secured by a graphical interpolation between observations lying most nearly contiguous to the standard air mass required. The large scale of the

[7] This statement follows from the universal practice which for small air masses assumes that m=sec. z. If any error is involved in the application of this equation to observations at the same station on widely different days, seasons, etc., an additional cause for fluctuations in derived values of the solar constant of non-solar origin, is introduced.

diagrams permitted logs to be read to four significant figures. Unerring fidelity to the original observations was the main objective in this classification of the data, which was necessary to permit of the mass statistical studies we now present.

Solar constant and the scatter of parent data.—Since all observations on different days *at the same air mass* are nearly errorless we may choose any air mass as a standard of reference. Assuming that sky conditions permit, there are several advantages in favor of high sun, i. e., small air mass, conditions. The intensities are highest and air mass most nearly constant; accordingly errors are least. The effects of atmospheric extinction and depletion are least and the effects of changes of solar intensity greatest.

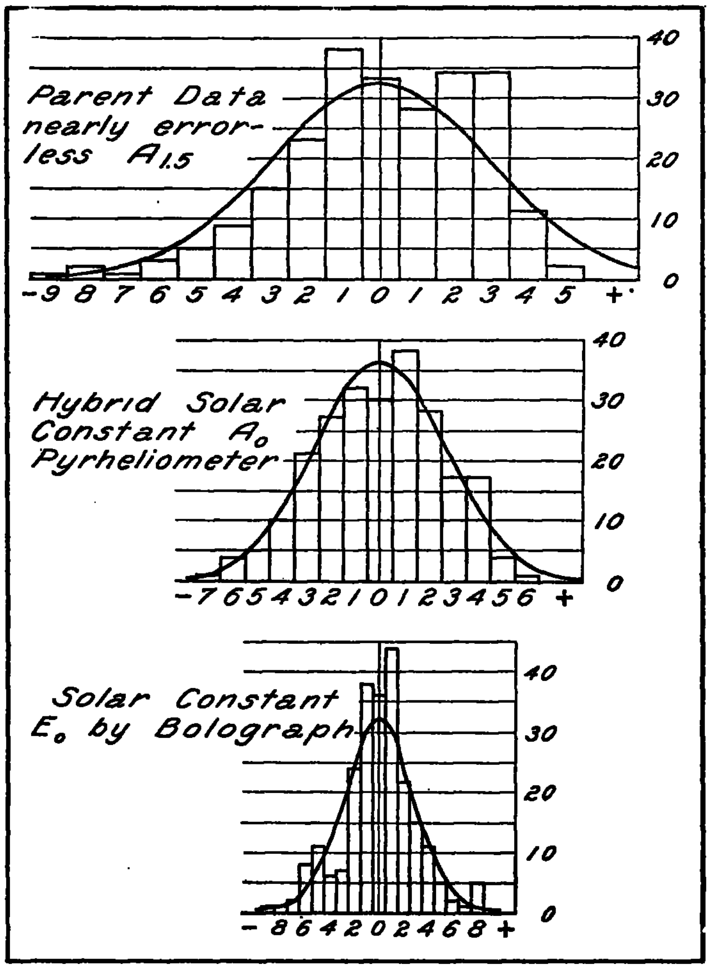


FIG. 2.—Diagram showing in comparison and contrast the frequency distribution of 239 daily values of the solar intensity: (1) The nearly errorless parent pyrheliometer values as observed at air mass 1.5; (2) the hybrid solar constant derived by straight-line extrapolation of the pyrheliometer observations to zero air mass; (3) the solar constant E_0 derived by the bolographic method

For these reasons we have chosen for analysis the data for air mass 1.5, and figure 2 shows the frequency distribution of the original nearly errorless observed facts and what happens when daily observations are extrapolated to zero air mass by equation (6) and by the operations of the bolographic computations.

The standard deviations, σ_t, for the three groups, $A_{1.5}$, A_0, and E_0 are nearly in the order 3, 2, 1, respectively. There is actually nearly 80 per cent more scatter in the hybrid solar constant A_0 than in the Langley bolographic value E_0 derived from the same parent data, represented by $A_{1.5}$. Since the bolographic process necessarily introduces a whole family of errors and variations all its own, which are not in the parent observations at all, and since the bolograph can not exclude those variations which *are* in the parent data, due to failure of the sun and the transparency of the air to remain constant during observations at different air masses, we are compelled to explain the 80 per cent greater scatter of log A_0 (± 0.01059 log unit) as compared with that of log E_0 (± 0.00594 same unit) as caused by variations due to polychromatic effects. If our reasoning is sound and the above proportion chargeable to polychromatic effects at a station like Calama, Chile, is physically too large, as seems to be the case, then some question arises as to the validity of the bolographic operations. However, no conclusion can be reached until we have had opportunity to extend the same analysis to other bodies of independent data.

Equation (17) applied to scatter at different air masses.—Equation (17) serves to separate the scatter due to change in transparency from one day to another from the remaining scatter, which can be due only to errors and to the sun. Putting T_m = the total scatter at air mass m due to all causes, equation (17) may be written

$$T_m = \sqrt{\sigma_{xi}^2 + m^2 \sigma_1^2} \qquad (18)$$

in which σ_{xi} is the combined variation due to errors and to the sun; that is, to all other causes than the day-by-day changes in transparency represented by $m\sigma_1$. We have six equations and two unknowns and the following table gives the results of a least square computation of σ_1 and σ_{xi}.

TABLE 1.—*Total variation, σ_t of 239 day-to-day pyrheliometer values for Calama at different air masses, with calculations of σ_1 and σ_{xi}.*

	1	2	3	4	5
Air mass m	Mean[1] value $\overline{A}_m$ log units	Standard deviation σ_t log units	Theoretical, no errors	Least square values, equation (18) Log units $m\sigma_1$	σ_{xi}
0 $\overline{E}_0$	0.28862	$\pm$0.00594	$\sigma_t = \sigma_1$	0	$\pm$0.01304
$\overline{A}_0$	.24377	$\pm$.01059			
$\overline{A}_s$	.24336	$\pm$.00744			
$\overline{A}_q$	.25728	$\pm$.00388			
1	------	------	$\pm 1.41\sigma_1$	$\pm$0.00640	Residuals.
1.5	.18076	$\pm$.01550	$\pm 1.80\sigma_1$	$\pm$.00958	−0.0085
2	.15568	$\pm$.01842	$\pm 2.24\sigma_1$	$\pm$.01277	−.0023
3	.11002	$\pm$.02525	$\pm 3.16\sigma_1$	$\pm$.01916	+.0341
4	.06770	$\pm$.02905	$\pm 4.12\sigma_1$	$\pm$.02554	+.0079
5	.02807	$\pm$.03377	$\pm 5.10\sigma_1$	$\pm$.03193	−.0204

[1] Mean values are designated by a superior bar.

Explanation of Table 1.—Column 1 contains the mean value of the log A_m derived from the several classifications for computing σ_t. Only 4 place logs were used but the 5th place is retained in means.

Column 2 contains for air masses 1.5 to 5 the measure of total scatter due to all causes as obtained directly from nearly errorless observations over a climatic interval of one year. These values are the parent data from which all information must be derived.

The several values at air mass 0 are results derived by various methods devised to compensate for the losses of solar intensity caused by atmospheric depletion. At the same time, each method unavoidably introduces greater or lesser variations of its own, which are not present at all in the parent data. These values will be discussed later.

Column 3 represents what we ought to get from errorless data on the assumption that the total observed variations at air mass 1 are caused to an equal degree by the day-to-day changes in solar intensity and air transparency.

The very small effect of solar changes at air masses 4 and 5 shows how little significance such observations have in revealing solar fluctuations.

Columns 4 and 5 give the values of σ_1 and σ_{xi} derived from a least square application of equation (18) to the data in column 2. Changes in air conditions between observations are of course purely relative matters. Hence it is permissible to assume that observations at air mass 5 are standard; that is, that all changes between observations at different air masses occur *after* the observation at air mass 5. Consequently, observations at air mass 5 as a group are nearly errorless. If now no changes whatever had occurred on any day between observations and the solar intensity had also remained constant throughout the year, then the value $\sigma_1 = \pm 0.03377 \div 5 = \pm 0.00675$ is the amount of change we should have found at air mass 1. It is significant that the least square value $\sigma_1 = \pm 0.00640$ is almost identically the same. The latter value we regard as the most exact measure we can get, from the parent data, of the scatter due to the true changes in atmospheric transparency from day to day.

The other value derived from the computation, by least squares $\sigma_{xi} = \pm .01304$, is the total variation which we ought to get at air mass 0 due to all the errors and variations in the parent data *except those caused by variation due to atmospheric depletion.* Those included are: (x) instrumental errors combined with variations due to solar and atmospheric changes between observations, and (i) solar changes from day to day.

Although this value $\pm .01304$, which represents the total variation at air mass zero, is larger than any other, it is perfectly valid, being large simply because equation (18) (not by previous design but simply in effect) extrapolates to zero air mass *all* the variations due to the failure of the air and the sun to remain constant between observations. When an observer makes this extrapolation by eye he distributes the variations among the observations at different air masses as his judgment dictates. Equation (18) gives us a *large scatter at air mass zero and a true measure of day-to-day variations in air transparency.* The observer's judgment, on the contrary, by giving different weight to the observation at different air masses, results in individual values for zero air mass which are inaccurate (although showing smaller scatter) because associated with an erroneous value of the coefficient of atmospheric transmission. The combination of these circumstances produces considerable negative correlation, for, almost without exception the variations in the solar constant value show an inverse relation to those of the coefficient of atmospheric transmission.

As a basis for discussing the most important feature in the table, namely, the wide scatter of the four values for air mass 0 (column 2), these are assembled in Table 2, together with σ_{xi} from column 5.

TABLE 2.—*Standard deviation σ_t due to all causes at air mass zero by different methods*

Symbol	σ_t	Remarks
$\bar{E}_o$	±.00594	Bolographically determined, includes all variations x and i in parent data and adds a family of errors of its own.
A_o	±.01059	Pyrheliometric values include all variations x and i in parent data and add errors due to extrapolation of polychromatic radiation by a straight line.
A_s	±.00744	Secured by taking half the difference between the intercepts at air mass zero of two least square straight lines fitted to points $\bar{A}_m + \sigma_m$ and $\bar{A}_m - \sigma_m$.
A_q	±.00388	Secured like A_s but the extrapolation to zero air mass effected by quadratic equation (7). See Fig. 1.
σ_{xi}	±.01304	Total variation due to errors and sun as evaluated by equation (18).

The wide difference and seeming contradiction between the values $\pm .00388$ and $\pm .01304$ are really consistent and easily interpreted. The value $\sigma_t = \pm .00388$ is a valid statistical datum of day-to-day variability, more free than any of the four others from the harmful variations due to unavoidable errors of various kinds. It is derived directly from the entire body of parent data without introducing appreciable errors of its own. It is the value we ought to secure if each day's observations could be extrapolated to zero air mass with no more error than applies to the mean values for the year, which are nearly free from the large errors affecting the extrapolated daily values.

The quadratic equations for the data $\bar{A}_m + \sigma_m$, $\bar{A}_m$ and $\bar{A}_m - \sigma_m$ designated by subscripts 1, 2, 3, are, respectively,

$$A_1 = 0.26217 - .0462m + .00122m^2$$
$$A_2 = 0.25728 - .0538m + .00158m^2$$
$$A_3 = 0.25442 - .0629m + .00218m^2$$

Quantitative result of analysis.—Using the minimum value of scatter found, $\pm .00388$, as least affected by terrestrial causes of fluctuations, and reducing the standard deviation σ to probable variation, we get the following mean value of the solar intensity for the year and the day-to-day fluctuation in calories and percentage, viz:

$$\bar{E}_o = 1.9436 \ \pm (.0117 = 0.60\%)$$

For the purposes of the above conversion of units it is quite immaterial what value of the solar constant we use. We take the mean value for the year, $\bar{E}_o$, as probably nearest the true mean and find a percentage variation of only ± 0.60, which is just about the order of accuracy of the recent observations of the Smithsonian Institution by the pyranometer.

The reader should remember that the value $\pm .0117$ calories is a measure of the probable departure from the mean annual solar constant of any daily measurement when freed to the highest degree from all kinds of errors. It is derived directly from pyrheliometer observations at air masses from nearly 1 to over 5 on 239 days at Calama, Chile, from July 27, 1918, to July 24, 1919. This minimum value is only two-thirds as large as the average variation shown by the bolographic reduction of the same parent data and is equal to the scatter of recent high grade observations by the pyranometer. Considering the great difficulty in securing extreme accuracy in any daily value of the atmospheric depletion of incoming radiation, this small total probable departure may well be nothing but unavoidable error. Nevertheless, small as it is, we are still justified in assuming that a part of it is caused by day-to-day changes in solar intensity. To be fair to both sides of the question, let the total variation be equally apportioned to errors and to the sun. The share of possible solar variation then becomes $\pm .0117 \div \sqrt{2} = \pm .0083$ calories.

There is no definite statistical evidence as yet that any of the total observed variations have a solar origin, and the foregoing possible amount of variation *by assumption* approaches the irrational, because we have no assurance whatever that the total $\pm .0117$ is the irreducible minimum. When it is possible to secure by standardized pyrheliometers alone their nearly errorless observations at n *widely separated and independent stations,* we must inevitably cut down the above value σ_x in the ratio of 1 to $\sqrt{n}$. Otherwise we approach the absurdity that the errors of the Calama observations are but a small fraction of the total variations found to be $\pm .0117$. It is quite in harmony with past experience to expect that the mean of only *three* stations making pyrheliometer readings as good as those at Calama will reduce this total to $\pm .0117 \div \sqrt{3} = \pm .0068$ calories, a *whole* variation which is now less than the *part* of the Calama variations assumed above to be of solar origin.

Here again final conclusions must be reserved until data from independent stations are available and until other studies now in progress are completed.

Annual periodicity in Calama solar constant values.—There seems to be no sufficient reason why there should be a twelve-month period in solar intensities. Especially is this true when such a period is correlated to a high degree with seasonal and annual states of the atmosphere induced by changes in vapor pressure, precipitable water, transparency, etc. The presence of such periods in values of the solar constant is prima facie evidence that there are present in those values important day-to-day errors of entirely terrestrial origin. No criticism or objection need be made to these relatively small errors when taken in connection with annual mean values of solar intensity, but the most serious objections are justified when it is insisted that the day-to-day variations in *derived daily solar constant values* should be accepted as fair representations of day-to-day and other short-time changes in *solar intensity*. The latter proposition is very far from having been proved. Doctor Abbot feels that his critics are too exacting and should not require impeccable observations. As one who has only praise, not criticism, to express for the splendid quality of Doctor Abbot's work as a whole, I want to make it clear that on my part at least, faulty observations are accepted as inevitable. What I desire is that every fault of the original observations be made clear, so that students may know with just what they are dealing. The great need is for a flawless interpretation of all the bits of evidence, whether for or against solar fluctuations.

The only way in which we can ever hope to get at the root of the matter is to bring into the foreground every known cause of fluctuations in the derived values. Under such conditions, no real solar variation could possibly escape detection and partial evaluation. Any other course implies a lack of confidence in the enormous power of modern statistical methods to reveal the secrets deeply hidden in large masses of homogeneous data.

Again, the point can not be too strongly emphasized that a whole year is the shortest climatic interval which can legitimately be employed in the analysis and adjustment of solar constant data. The atmospheric states of transparency, dustiness, water vapor content, convection, etc., have a regular cycle of their own completed only in the round of a full year, and these states exert such a direct and profound influence upon the values of the solar constant that it is futile to hope that a few months' observations are free from highly important systematic and semiconstant errors, or that empirical corrections, reductions, reduction factors, function values, etc., can be satisfactorily evaluated in less than a year if at all.

Figure 3 tells in such a graphic way its own story of the annual periodicities in the Calama observations that little explanation is required. The number of observations per month averaged 20 and were distributed as follows:

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
17	20	15	26	25	17	12	25	18	24	22	18

The distribution leaves very little room for criticism as to the realness of the annual features shown by the monthly means.

Theoretically the extrapolation of daily observations to zero air mass is supposed and expected to faithfully *exclude* the effects of purely terrestrial and atmospheric states. The diagram shows at once to the eye, without correlation coefficients or other quantitative measurements that the theory and expectation are satisfied only in part. Actually, each terrestrial feature of annual periodicity, whether in transparency, water content of the air, or intensity for a given air mass, *is more or less faithfully extrapolated*, and is not excluded from solar constant values. Nevertheless, it is gratifying to point out that the monthly values of E_0 for 1918 to 1920 at Calama are more nearly free from annual periodicity than any other group of annual values published either before or since. The details of this matter will be presented in Section V.

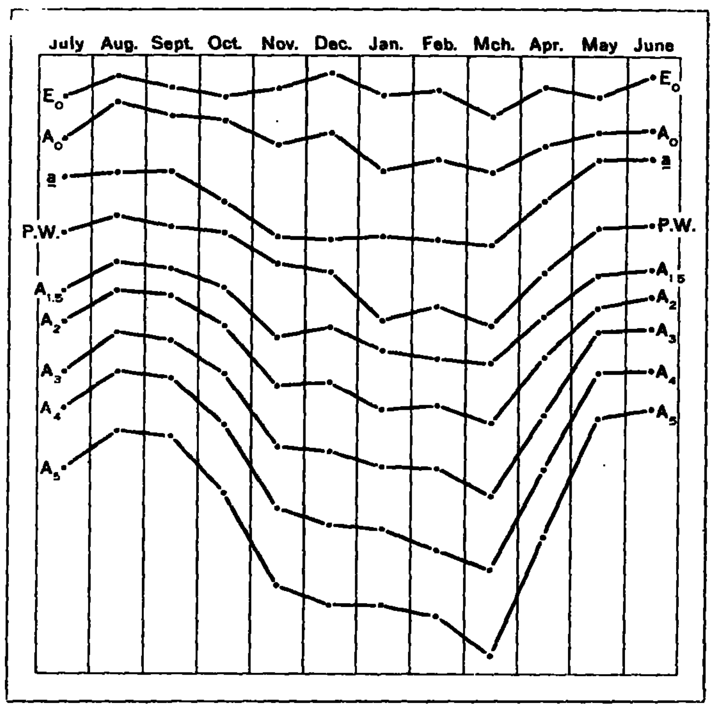


FIG. 3.—Monthly mean solar intensities as observed at different air masses by the pyrheliometer at Calama on 239 days, also values extrapolated to zero air mass, including values of air transparency *a* and atmospheric precipitable water P. W. (plot inverted), all showing correlation and 12-month periodicity

If we were seeking statistical evidence of the change of solar intensity from one month to the next, instead of from one day to the next, we could point to the systematic increase in the value of E_0 culminating in December, with subsequent decline to a minimum in March. The December feature fairly satisfies the conditions of equation (18), but the relations of the companion feature of the March minimum to the parent data are entirely opposed to those prescribed in that equation.

The hypothesis of solar variation in this particular instance is further invalidated by the high negative correlation of E_0 with the transparency and water content of the air. No one can say what the sequence of daily values of E_0 would show if freed more completely from these sources of error.

III. ANALYSIS OF CALAMA DATA BY CORRELATIONS

The statistical device of the correlation coefficient must not be neglected in our search for the facts about day-to-day solar variations.

A graphical tabulation of the significance of such statistical indices derived from the analysis of observations passing progressively from assumed simple and more or less ideal conditions to actual conditions affected by errors, seems to be the most forceful way of presenting what is desired in this section. To aid those who may not be versed in the subject of correlations, this tabulation may be prefaced by a brief statement of a few fundamental principles.

Errorless observations of two quantities a and A, which are known to be in linear functional relation, will show perfect correlation, ± 1.00, provided no other cause than a can produce changes in the values of A. If a second cause, I, wholly independent of a and A, can also cause observed values of A to change, and if I is just as potent as a in producing changes in A, the correlation coefficient between a and A will be $\sqrt{(\frac{1}{2})^2+(\frac{1}{2})^2}=.71$. If I dominates, then the correlation between a and A will be less than 0.71, according to the relative potency of I and a. If a dominates, the correlation will be higher than 0.71. A correlation of 0.91 for errorless values of a and A would imply that a is nine times as potent as I in causing changes of A.

Errors of measurement are of course causes of variation and produce definite effects on correlation coefficients. The potency of entirely fortuitous errors, if large, compared with that of a physical cause like a, may change a ± 1.00 correlation between errorless values of a and A to a small coefficient of, say $\pm .10$, more or less. If the number of values in correlation is sufficiently large, the sign of a true correlation will not be changed by errors, but if the number is small, or if the errors are partly systematic, their presence may not only reduce the size of the real correlation but may even change the sign. Such effects of errors will be the same, regardless of whether one or several causes in combination produce the correlations.

Before reading what follows, Table 3, on page 297, should be carefully examined.

Discussion of Case V of Table 3.—This case deals with the actual pyrheliometer and other observations made at Calama July, 1918, to July, 1919. See Figure 1 and the discussion of scatter values already given in Section II.

In considering correlations between derived solar constant values it must not be forgotten that every cause of variation is known and that, excepting purely fortuitous errors, the causes are united in known functional relations, mostly linear—all of which makes the results presented in Table 4 of the highest significance.

The essential features of Table 4 appear in the first four lines which should be compared with corresponding data in Cases I, II, and III.

TABLE 4.—*Correlations between logs of observed pyrheliometer values adjusted to standard air masses 1.5 to 5, and values of solar constants E_0, A_0 and fair transparency a, also precipitable water P. W.*

		a	E_0	A_0	$A_{1.5}$	A_2	A_3	A_4	A_5
1	*p. w.*	−0.59	−0.33	−0.56	−0.74	−0.79	−0.80	−0.69	−0.55
2	a	--------	−.50	−.25	+.49	+.53	+.64	+.69	+.51
3	E_0	--------	--------	+.69	+.48	+.56	+.45	+.55	−.18
4	A_0	--------	--------	--------	+.61	+.67	+.61	+.53	+.65
5	$A_{1.5}$	--------	--------	--------	--------	+.88	+.82	+.72	+.51
6	A_2	--------	--------	--------	--------	--------	+.84	+.73	+.48
7	A_3	--------	--------	--------	--------	--------	--------	+.70	+.40
8	A_4	--------	--------	--------	--------	--------	--------	--------	+.59

The high negative correlation of E_0 and A_0 with a and $p. w.$ is evidence of the grave fault in all the solar constant values, which almost without exception show a considerable negative correlation with atmospheric transmission coefficients. Zero correlation should be found, because real day-to-day solar variations can not be related in any direct way to atmospheric transparency or water vapor. The correlation E_0 and a, -0.50, as compared with -0.25 for A_0 and a, does not necessarily signify that A_0 values are better than E_0, because, as pointed out in connection with Figure 2, the scatter of A_0 is 80 per cent greater than that of E_0. We may therefore infer that fortuitous variations in A_0 due to polychromatic effects, which are absent from E_0, reduce the correlation of A_0 and a below what it would otherwise be.

The remaining correlations in the first two lines are entirely rational but would be much higher except for fortuitous variations due to errors.

Lines 3 and 4 of the table tell a very definite story. Errorless values of E_0 and A_0 should show a high correlation unless the fortuitous differences between them due to polychromatic radiation, as distinguished from all other causes of error, are themselves inherently large. This is a matter deserving fuller investigation. The table shows a coefficient of $+0.69$, which interpreted by Dines'[8] law means that only 48 per cent of day-to-day variation in these two values of the solar constant, which are derived from the same parent data, occur in synchronism.

Still greater interest attaches to the correlations with intensities at the different air masses, which will now be discussed.

Sun constant from day to day.—All variations in E_0 and A_0, except those in A_0 due to polychromatic radiation will now be fortuitous errors and therefore uncorrelated with intensities at different air masses. The slightly larger coefficients in line 4 over those in line 3 may be caused by a functional relation between variations due to polychromatic radiation and change in transparency of the air from day to day.

Sun variable; no errors.—The coefficient E_0 and $A_{1.5}$ $+.48$ may be interpreted to mean that day-to-day changes in air transparency cause three times as much variation at air mass 1.5 as that caused by the sun; that is, the sun causes about one-fourth the whole. On this basis, the correlation a and $A_{1.5}$, line 2, should be 0.88 $=\sqrt{1-(48)^2}$ instead of 0.49, and all the remaining coefficients in lines 2, 3 and 4 should be radically different from what they are. There is no escape from the conclusion that the assumption of day-to-day solar variation as great as one-fourth the total observed at air mass 1.5 is entirely invalidated by the mutual correlations of Table 4.

Of the evidence in lines 2, 3, and 4 the most rational interpretation is, that since all the observations are permeated with errors the greater part of these errors are faithfully extrapolated to zero air mass and there appear as variations in the values of the solar constants E_0 and A_0.

The correlations in column A_5 are all noticeably low. We are inclined to regard this as wholly due to the accumulation of considerable relative errors in the data in this part of our original values. The errors are not inherent in the observations separately, but, owing to atmospheric changes, observations at different air masses are erroneous relative to each other. In the practical work of extrapolating, the observer habitually

[8] Meteorological Magazine, Feb., 1921.

gives more weight to the data from smaller air masses, thus making the relative errors large at air mass 5 and producing lower coefficients of correlation than should be the case.

The correlations between intensities observed *within* the atmosphere are quite rational, but seemingly low. This is to be expected when we remember that a value for each individual air mass is nearly errorless in itself, but when compared with the value for another air mass on the same day the relative errors due to atmospheric changes (which often take place in only a few minutes) become serious and cut down correlation coefficients to relatively low values.

Weather Bureau pyrheliometer observations at Washington, D. C.—Dr. H. H. Kimball has kindly assembled for me the logs of intensities and values of *a* for observations on a total of 105 of the clearest days possible for Washington between the dates December 17, 1914, and June 26,

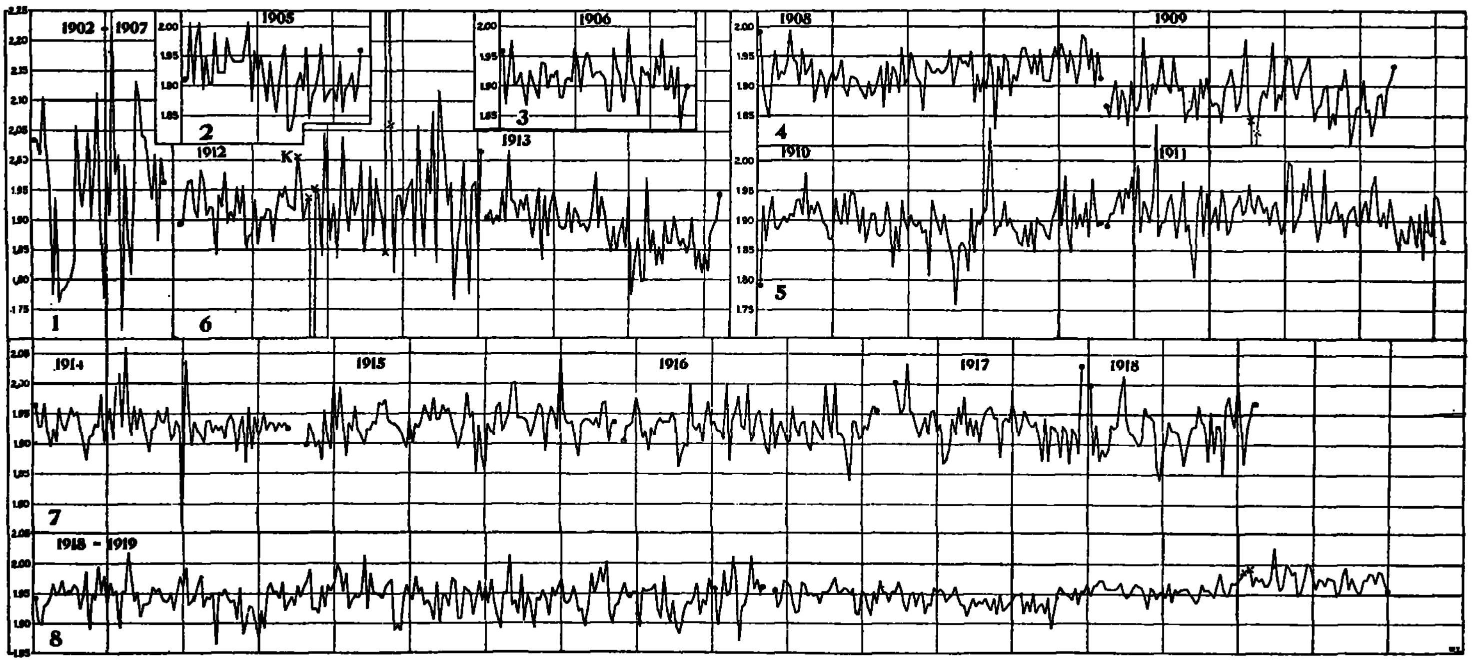


FIG. 4.—Diagram of nearly 2,000 observed values of solar constant as determined by the Smithsonian Institution from the beginning of observations in 1902 to the end of 1919 at stations Washington, D. C., Mount Wilson, Calif., and Calama, Chile

1925. These readings range by half air mass intervals from 1.5 to 5 and were read from a smooth curve run through numerous observations by means of a spline, a method which served also to give the extrapolation to zero air mass. The value of *a* was deduced from the slope of a *straight* line generally passing through the observations nearest air masses 1.5 and 4. The exhaustive analysis of these observations is not yet completed, but it is noteworthy that the correlation of A_0 with *a* came out exactly zero, as we like to have it.

Among the 105 days there were 59 on which values of E_0 were found by the Smithsonian Institution at Mount Wilson or one other of its stations. The correlation between these few values was +0.13. While the number of variants and the size of the coefficient is very small, the latter signifying an efficacy of only $\frac{1}{60}$, it has the right sign for a very small solar variation. Here again final conclusions must await confirmation from other bodies of data.

The foregoing Sections II and III are submitted both as an example of how a body of homogeneous observations of the pyrheliometer alone may be analyzed and interpreted in terms of day-to-day changes in solar intensity and also to show how weak the statistical evidence still is for any appreciable changes of solar intensity.

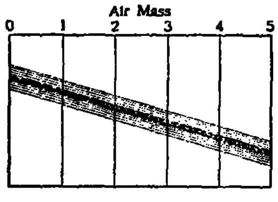
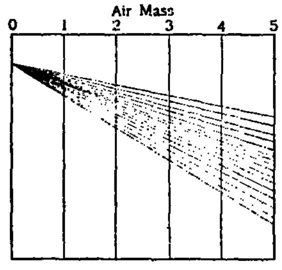
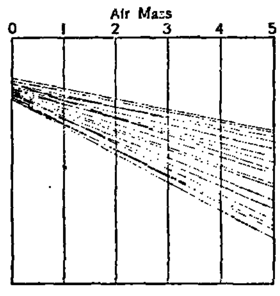
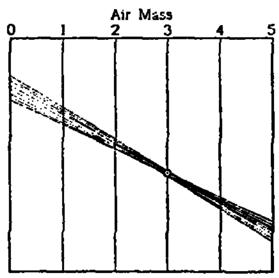
In closing these sections I wish especially to make it clear that I disclaim any intimation that the quantity A_0 is anything more than the name I have given it connotes, namely, a *hybrid solar constant.* I do insist, however, *that the pyrheliometer measurements, including values of A_0, contain within themselves all the observational evidence we have of day-to-day or other short-period variations in solar intensity.* The bolograph and pyranometer, either separately or in combination, are powerless to take from or add to the total intensity registered by the pyrheliometer, *except in the form of an errorless extrapolation of the observed value to zero air mass.* The fallibility of human measurements is known to be such that errorless extrapolation to zero air mass is impossible, therefore arbitrary empirical instruments like the bolograph and pyranometer only add their own inherent error to those of the parent data. Therefore, again, any variations in the extrapolated data that can not be absolutely shown to reside in the parent data and to be not due to error in those data, must be ascribed to errors of the subsidiary instruments and methods. *Such variations can not possibly be ascribed to the sun unless and until they can be identified in the parent data.*

IV. GENERAL EXAMINATION OF THE VARIABILITY OF ALL VALUES OF THE SOLAR CONSTANT BY BOLOGRAPH AND PYRANOMETER

A general conception of the whole question of variability of solar constant values is most readily gained by a careful inspection of Figures 4 and 5, which show in consecutive order practically all observations published from 1902 to November, 1924. The reader is asked to follow closely in Figure 4 the groups of data numbered consecutively 1, 2, 3—8, noticing especially the great increase in range of values in 1912, when the explosive volcanic eruption of Mount Katmai caused a notable increase in the scatter of values due to dust in the high

TABLE 3.—ANALYSIS OF THE EFFECTS ON DERIVED VALUES OF THE SOLAR CONSTANT EXERTED BY DIFFERENT INDEPENDENT CAUSES

(The relations between measures of scatter, and the mutual correlations among all the variables, are shown in the columns headed "scatter" and "correlations". Diagrams showing extrapolation to zero air mass represent the mathematical relations for any beam of monochromatic radiation rigorously, and for polychromatic radiation approximately.)

Assumed conditions			Effects as reflected in observations		
Case No.	Solar	Terrestrial	Extrapolation to zero air mass	Scatter	Correlations (All positive except as otherwise indicated)
I	All changes are caused by sun alone. Scatter $= \sigma_1$	Conditions and instruments ideal. No errors or changes of any kind. Transparency of air constant from hour to hour and day to day indefinitely. Transmission coefficient = a		Scatter of observed values at all air masses identical and same as in sun $= \sigma_1$	See Case I correlations below
II	Sun and I_0, E_0 A_0 absolutely constant.	No errors of any kind. Conditions still ideal as in I, except that while air transmission a remains constant during one day's observations a changes irregularly from one day to the next.		The a lines of extrapolation focus to an exact point at zero air mass. Scatter there equals zero; elsewhere as below: Air mass — Scatter 0 — 0 1 — σ_1 2 — $2\sigma_1$ 3 — $3\sigma_1$ 4 — $4\sigma_1$ 5 — $5\sigma_1$	See Case II correlations below
III	Cases I and II combined, two independent and equal causes of variation. Sun constant during observations, but changes intensity irregularly from day to day. Scatter $= \sigma_1$	No errors or other causes of variation, except transparency, which remains constant during observations but changes irregularly from day to day. Scatter $= \sigma_1$		Total variation now reflects changes due to both the sun and day to day changes in transparency. By the law of propagation of variations, scatter becomes $\sigma_m = \sqrt{\sigma_1^2 + (m\sigma_1)^2}$ Air mass — Scatter $\sigma_i = \sigma_1$ 0 — σ_1 1 — $1.41\sigma_1$ 2 — $2.24\sigma_1$ 3 — $3.16\sigma_1$ 4 — $4.12\sigma_1$ 5 — $5.10\sigma_1$	See Case III correlations below
IV	Solar conditions constant in every particular.	All variations due solely to errors, caused, first, by fallibility of observer and his instruments. These are the errors within the observatory. Second, the failure of the transparency of the air to remain constant during low and high sun observations. This is *the atmospheric cause of error. To simplify* representation, assume the transparency of the air to be the same day after day at the time observations are made at air mass 3, and that before and after this time the transparency changes each day in an irregular but natural way by hazings and clearings.		The values secured by extrapolation to zero air mass *and the scatter in Case IV can* hardly be distinguished from those allowed as possible for Case I, representing pure solar variation. At other air masses the scatter *increases above* and below air mass 3. The little circle at air mass 3 represents the possible error of the pyrheliometer alone. Every *observation under the assumed* conditions falls within the circle.	See Case IV correlations below
V	This is the actual natural problem. Four entirely independent causes, three of which are in strictly functional and linear relations, produce variations in observations and results: namely, (1) air mass, the only cause under control; (2) change in air transparency from one day to the next; (3) possible solar changes from day to day; (4) all errors in combination, classing change in transparency and solar intensity during observations as a source of error. Errors produce variations which are not in functional relations, but often of semisystematic character, with seasonal and annual features. The detailed discussion of the actual data will be made in the following text.				

Correlations, Case I:

	E_0	A_0	A_1	A_2	A_3	A_4	A_5
a		0	0	0	0	0	0
A_0			1.00	1.00	1.00	1.00	1.00
A_1				1.00	1.00	1.00	1.00
A_2					1.00	1.00	1.00
A_3						1.00	1.00
A_4							1.00

Correlations, Case II:

	E_0	A_0	A_1	A_2	A_3	A_4	A_5
a		0	1.00	1.00	1.00	1.00	1.00
A_0			0	0	0	0	0
A_1				1.00	1.00	1.00	1.00
A_2					1.00	1.00	1.00
A_3						1.00	1.00
A_4							1.00

Correlations, Case III:

	E_0	A_0	A_1	A_2	A_3	A_4	A_5
a	0	0	.77*	.91	.36	.98	.99
A_0			.63*	.42	.28	.20	.14
A_1				.88	.82	.80	79
A_2					.95	.93	.92
A_3						.98	.97
A_4							.99

* See remarks.

Correlations, Case IV:

	E_0	A_0	A_1	A_2	A_3	A_4	A_5
a	−.50±	−.50	±0	±0	±0	±0	±0
E_0		±0	±0	±0	±0	±0	±0
A_0			±0	±0	±0	±0	±0
A_1				±0	±0	±0	±0
A_2					±0	±0	±0
A_3						±0	±0
A_4							±0

REMARKS FOR TABLE

CASE I.—Scatter at all air masses must be the same. Transparency and solar constant must be uncorrelated because the transparency of the earth's air cannot cause solar changes, and solar changes do not in any direct and instantaneous way change atmospheric transparency. Correlations between all intensities = 1.00.

CASE II.—No cause for variation except change in transparency from day to day. Scatter directly proportional to air mass for errorless observations. Correlations possible only between intensities at different air masses, and all = 1.00.

CASE III.—Spontaneous and unrelated changes in sun and atmospheric transparency now the sole cause of variation of intensities at the different air masses and all are measured without error. The diagram and table of correlations represent (but not to scale) 101 errorless observations satisfying the conditions assumed, namely, (1) that the sun spontaneously undergoes variations about equal to the total variations observed at Calama from July, 1918, to July, 1919, and (2) that the transparency likewise spontaneously changes from day to day to such degree that intensities at air mass 1 are statistically identical with those assumed for the sun, but wholly uncorrelated thereto. The 101 pairs of variants A_0 and a are drawn from a bowl of Gaussian numbers. The "errorless observations" are the intensities at air masses 1 to 5 derived by a reverse extrapolation calculated with rigorous accuracy on a Marchant Calculator. Correlations by Clough's method. The two drawings of fortuitous numbers should have shown equal correlations, $1 \div \sqrt{2} = .71$ The slightly different values, column A_1, .77 and .63, *mean that the random drawings failed slightly to conform to theory.*

CASE IV.—All day to day variations in this case are caused solely by the aggregate of all errors, the effects and evaluation of which is, of course, the moot question of this whole subject. To make any adequate evaluation of such errors, with their annual and seasonal characteristics, requires access to the original observational records comprising one or more full years, so as to include all kinds of atmospheric states.

Case IV assumes that the sun is constant and that at the time the air mass is 3 the air transparency is exactly the same on each day, but that the transparency changes in a natural way during the period covered by observations at other air masses. The peculiar assumption is only to aid in visualizing the effects of errors. *The one value at air mass 3 is almost errorless,* being affected only by the small error of the pyrheliometer. The straight lines of extrapolation to zero air mass will cross above and below A_3 so as to best fit this and the other values for the day, resulting in the diagram as shown. Any doubt that conditions like these cause large variations in the derived values of the solar constant is dispelled by a critical acquaintance with actual daily observations and such mass diagrams of observations as Fig. 1 for Calama for the year July, 1918, to July, 1919. The scatter of values at air mass zero is drawn to the same scale as in Cases I and III, to represent very nearly the observed variations at Calama during 1918–19, as if all of them were due to errors alone. Obviously, only a part of the observed variations can be ascribed to the Sun, because errors can never be zero.

In the absence of an actual evaluation of X we cannot assign numerical values to the correlations, except that the well-known negative correlation between E_0 or A_0 and a will certainly be —.50, more or less. The other correlations, being caused more or less accidentally (waiving a possible habitual tendency of the air to haze up instead of clear up), will have small positive and negative values designated by the symbol ± 0.

strata. Smaller scatter attended the subsidence of this dust during the next two years, followed by a fairly steady state of the record during the five years until 1918, when the new station at Calama, Chile, began observations under improved atmospheric conditions. We notice here in the first half of Group 8 an appreciable drop in the scatter. Finally, in 1919 (latter half of Group 8) a new type of observing by means of the pyranometer led to a notable further drop in scatter.

Passing to Figure 5, Group 8 of Figure 1 is repeated, with later values *on an enlarged vertical scale* (lines 1, 2), followed by the sequence of all published values to November, 1924, on the same scale. All pyranometer readings show smaller scatter than bolographic observations.

The unaided mind is incapable of appraising the real significance and relative importance of these different amounts of variability in the different groups of data, but, happily, the science of statistics gives us several accurate measures of the varying degrees of scatter which such observations show.

The index of scatter best suited to present purposes seems to be the *probable error or variation* of a single daily result, expressed as a percentage of the average value of the solar constant which has nearly the same value, 1.94 calories per square centimeter per minute, for all groups.[9]

On this basis Figure 6 shows the scatter of the several groups of data shown in Figures 4 and 5.

Prior to the opening of the station at Calama, the very best observations showed a scatter of about 1 per cent. With the exception of the observations made in the summer of 1908 at Mount Wilson, the diagram tells us that up to 1911 the best observations show a scatter of about 1.3 per cent. Then came 1912 when Mount Katmai threw great quantities of dust into the high strata of the atmosphere, increased general atmospheric turbidity and caused the scatter to increase to fully double the best previous value, or over 2 per cent. This large

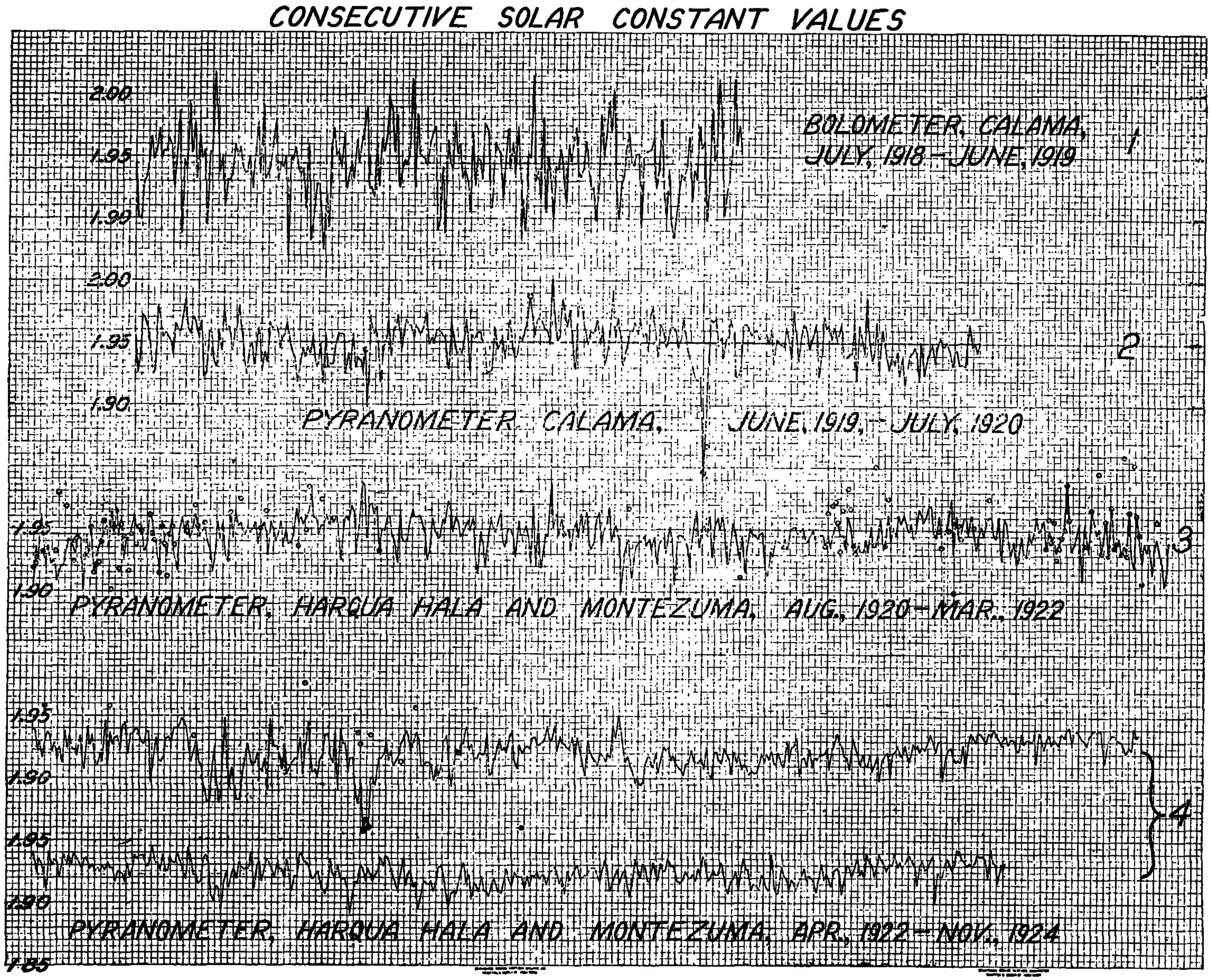


FIG. 5.—Consecutive values of the solar constant as observed at stations in South America from July, 1918, to November, 1924

[9] To remove any possible uncertainty concerning this measure of scatter we give the formula for its derivation.

$$e = \frac{.6745}{E_0}\sqrt{\frac{\Sigma v^2}{n-1}} = \frac{.6745}{E_0}\sigma \text{ when } n \text{ is large}$$

Σv^2 is the sum of the squares of the departures from E_0, the average value of the solar constant for a group. The standard deviation, $\sigma = \sqrt{\frac{\Sigma v^2}{n}}$, can often be used with convenience and accuracy.

index of variability gave place slowly in the four following years to low values, just over 1 per cent, for the years 1915 and 1916. The index of scatter then increased again to fully 1.3 per cent in the year 1918. Shall we say this increase was caused by greater solar activity or rather due simply to poor observing conditions at Mt. Wilson and wholly unavoidable errors of measurements? The latter conclusion is the correct one, *because the station at Calama had been put in operation and its observations for this same summer of 1918 show a decidedly smaller index of scatter and variability than ever before obtained.* Finally, even this small scatter of less than 1 per cent was cut very nearly in half, in the middle of the following summer, by the introduction of the pyranometer method of making observations. Happily, we have bolographic or long method observations for the first half of 1919, showing a scatter of just under 1 per cent, whereas the values by the new method for the latter half of 1919 show a scatter of only 0.52 per cent. Of course this change in scatter can not be explained by a sudden subsidence in solar variability coincident with the beginning of observations by the pyranometer, but is best explained as a reduction in the errors of measurement.

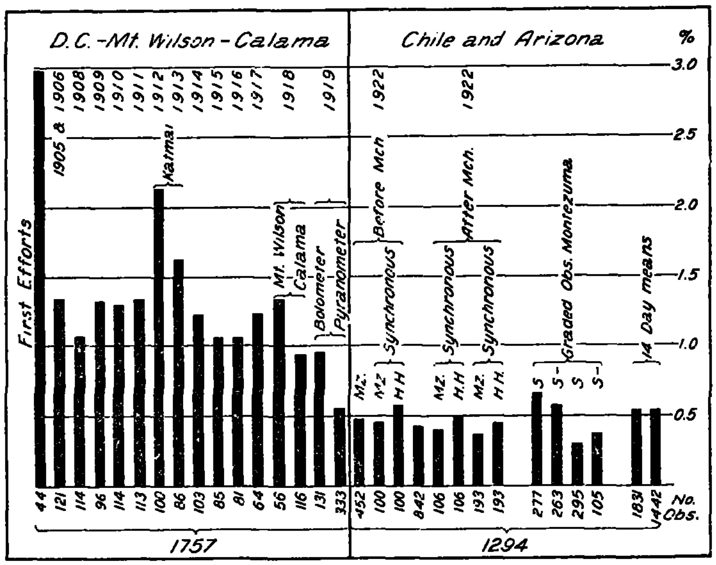


FIG. 6.—Probable percentage variation, due to all causes, of daily values of the solar constant, showing the values by groups from the older, more variable to the modern, less variable

Summing up the one outstanding result obtained by the assiduous program carried out up to 1919, it is this: *Atmospheric turbidity is the main cause of a large scatter in solar constant values, which scatter by the best stations and best methods was reduced to only 0.52 per cent by the close of 1919.*

Passing to the second part of Figure 6, representing the probable variation of several groups of data obtained at Montezuma and Harqua Hala during the years 1920 to 1924, we find in general a still further reduction in scatter. Thus 452 solar constant values spread over a period of nearly 20 months prior to April 1, 1922, show a scatter of only 0.49 of 1 per cent. Again, a group of 100 synchronous observations at the two stations within this period shows a scatter of 0.45 per cent for Montezuma and 0.59 per cent at Harqua Hala. Then, again, a large group of 824 highly comparable observations at both stations made after March, 1922, and covering a period of 32 months, gave a scatter of only 0.41 of 1 per cent. In these results we see the scatter going lower and lower as observations increase in number and methods are still further refined. During this same period we have two groups of synchronous observations at the two stations. One with 106 observations which show a scatter of 0.40 and 0.50 per cent, respectively. The other group comprises 193 observations with a scatter of 0.38 and 0.44 per cent. The smaller scatter in each case applies to Montezuma. It is hardly necessary to ask why this difference in scatter at the two stations. It is obvious that the variability of the sun can never be greater at one station than at another, especially for stations on nearly the same geographic meridian. It is equally obvious, however, that errors of measurement may differ greatly at the two stations, and that solar variability can not be greater than the least variability at the best station. That is, solar variability can not have been greater during the 32 months over which the 299 synchronous observations were spread than the two small values of scatter, 0.40 and 0.38 per cent at Montezuma.

We can not claim that even now these small measures of scatter represent mostly solar variability, because that involves the impossible assumption that each of the 299 observations was nearly 100 per cent perfect. There is no rational interpretation of the mute evidence presented by this great body of data except to recognize that both the large and the small day-to-day fluctuations in the value of the solar constant have always been largely, if not wholly, due to variations in the unavoidable errors of observation resulting chiefly from the ever-changing turbidity of the atmosphere.

A further evidence of the extreme smallness of possible solar variability may be added.

Doctor Abbott has classed each of the observations as made, into grades designated S, $S-$, $U+$, and U, meaning *satisfactory, nearly satisfactory, rather unsatisfactory,* and *unsatisfactory.* Though we do not believe that the grading of observations by arbitrary methods is likely to be wholly satisfactory, the results are accepted at face value and from an analysis of four groups we find as follows:

No. 1 comprises 277 S observations, covering about 34 months after August 3, 1920, and therefore includes the secular changes and drop in 1922, which doubtless causes the scatter, 0.66 per cent, to be larger than it would otherwise be.

No. 2 comprises 263 comparable $S-$ observations, covering the 52 months beginning August 1920 and showing the smaller scatter of 0.62 per cent. We should expect the observations graded as less satisfactory to show a larger scatter, while the actual result is just the reverse. Of course this is only one group of results.

No. 3 comprises 295 S observations, covering a period of about 18 months, beginning June 1, 1923. The Montezuma station was working very uniformly during all this period and observations of some kind were obtained almost every day. These 295 values show a scatter of only 0.30 per cent. Think of it: Over a total period of about 18 months, less than one-third of one per cent for all causes of variation!

Group No. 4 comprises 105 $S-$ observations, all interspersed among the S observations, and shows a scatter of 0.38 per cent.

We see from all the foregoing that the better our observations become, the smaller and smaller becomes the total variation due to all causes, and hence increasingly smaller must be the part which can be truly ascribed to solar changes.

Suppose real day-to-day changes of solar intensity of consequential magnitude actually occur. These can be statistically measured and represented by the symbol σ_i and such variations are related to errors of observation by our basic equation

$$\sigma_t = \sqrt{\sigma_i^2 + \sigma_x^2} \qquad (9)$$

We must recognize the inexorable consequences which flow from equation (9). Observations for more than 20 years have been giving us values of σ_t which have been growing smaller and smaller as better instruments, better methods, and better observing stations have been employed. Over all this period the day-to-day solar fluctuations σ_i if existent at all in consequential magnitude, have stood as an obstacle to diminution in the value σ_t. That is, σ_i is the irreducible minimum which σ_t approaches asymptotically as σ_x becomes zero. With σ_x still of finite value we have reached a low value of σ_t for the general run of recent observations of about ± 0.50 per cent more or less. This is now the total daily variation which we are required to apportion between errors and solar changes by means of equations (10), (11), (12). Before doing this we will first examine the whole body of data for annual periodicity.

V.—THE 12-MONTH PERIOD IN SOLAR CONSTANT VALUES FOR NORTHERN AND SOUTHERN HEMISPHERES

In this section will be shown the serious extent to which even the monthly mean values of the solar constant are systematically impressed with annual features associated with summer and winter states of the atmosphere. If monthly mean values, often based on observations for a period of several years, are subject to systematic terrestrial influences, how much more serious must be the everchanging atmospheric effects upon single daily determinations.

This analysis embraces practically all observations from 1905 to 1924. The 246 bolographic observations at Calama for the year July, 1918, to July, 1919, prove to be the best observations ever made for a single year, either before or since; that is, as a group they are most free from annual periodicity. Unfortunately, *frequent* daily bolographic observations terminated with the introduction of the pyranometer, beginning July, 1919. However, a total of 70 determinations of E_o were secured by the pure [10] bolographic method during the year July, 1919, to 1920. These 70 observations are fairly well distributed through all the months, averaging from 2 to 9 days per month, and appear to be of a high quality though limited in number; therefore I have combined all observations in both years into mean monthly values. Whether by accident or not these 316 daily values as a group show decidedly the smallest systematic seasonal effects of all the groups of data, large or small, yet examined. It is significant that the observations were secured by the pure bolographic method at a single station.

[10] The adjective *pure* is used occasionally to allude to bolographic observations carried out rigorously in accord with Langley's basic idea, giving a result designated E_o. A correction for water vapor was applied to all such results at Mount Wilson, giving a supposed superior value designated E'_o. These values show a greater annual period than any others.

TABLE 5.—*Monthly mean values of the solar constant at various times and stations from 1905 to November 1924*

	x	From 1905 to 1920					From July, 1919, to November, 1924			
		Mount Wilson, Calif. (omitting Katmai, years 1912 and 1913), after June, 1912			Bolograph Calama, July, 1918, to July, 1919		Pyranometer			
							Calama, July, 1919, to July, 1924		Harqua Hala, October, 1920, to October, 1924	
		Number months or years	E_o	E'_o	Number days	W. M.	Number days	W. M.	Number days	W. M.
April	0				35	1.9483	110	1.9310	68	1.9252
May	1	4	1.9132	1.9348	32	1.9414	85	1.9316	90	1.9312
June	2	11	1.9225	1.9459	25	1.9519	96	1.9276	88	1.9222
July	3	13	1.9159	1.9435	13	1.9350	99	1.9366	43	[a]1.9247
August	4	13	1.9160	1.9446	35	1.9539	96	1.9334	28	[b]1.9277
September	5	13	1.9139	1.9375	25	1.9416	90	1.9390	68	1.9240
October	6	11	1.9145	1.9344	30	1.9374	88	1.9398	76	1.9348
November	7	4	1.9080	1.9310	27	1.9395	93	1.9420	65	1.9378
December	8				27	1.9558	79	1.9392	45	1.9378
January	9				22	1.9430	77	1.9454	71	1.9388
February	10				22	1.9465	62	1.9376	67	1.9328
March	11				23	1.9429	105	1.9324	55	1.9262
Total		*Months* 69			316		1,080		764	

[a] July, 1923, missing. [b] August, 1923, missing.

TABLE 6.—*Constants and results of harmonic analysis of data in Table 5*

[$y = E_o + c \cos(\theta - \varphi)$ Epoch of origin Apr. 15]

Station	Curve	Annual mean calories E_o	Amplitude c	Phase constant φ [a]	Remarks
Calama, 18–20	1	1.9448	0.0016	0.916	316 pure bolographic values.
Calama, 18–19	1	1.9452	.0042	.851	246 of above values, curve in broken line.
Mount Wilson	2	1.9123	.0058	.249	69 months of summer observations, May to November.
Mount Wilson	4	1.9335	.0109	.271	Parent data identical with curve 2.
Calama, 19–24	3	1.9363	.0061	.616	Pyranometer data over 5 years, 1919 to 1924.
Harqua Hala	5	1.9308	.0072	.658	4 years' observations.

[a] The phase constant φ is not given in the customary angular units but in a number representing the fractional part of the length of the period, thus easily fixing the phase position of the maximum. With the origin at April 15 the maximum for phase constant $0.916 = 12 \times 0.916 = 11$ months after April 15, viz, March 15. Similarly, $360° \times 0.916 = 329.°76$ angular units.

The monthly and general annual means upon which Figure 7 is based comprise such a large body of representative data that final values are tabulated for permanent reference in Table 5. We also give in Table 6 for reference purposes the constants of the harmonic analysis of the data in Table 5. These tables and diagrams present in highly concentrated form the testimony of fully 3,000 daily observations covering a period of work of nearly 20 years. Each monthly value we employ is, with rare exceptions, the mean of many daily values. Moreover, our final results do not depend in any material way upon any particular monthly value. The striking harmony and consistency in the results (Harqua Hala excepted) are the combined testimony of the entire mass of homogeneous statistical numbers.

Discussion of Figure 7.—The sequence of monthly means in No. 1 shows large variations above and below the annual mean, but the amplitude of the least square sine curve, continuous line, is so small in relation to its obviously large probable error that the mathematical result should be interpreted as no annual period at all; that is, these monthly mean values of the solar constant on the whole are, as they should be, practically free from systematic terrestrial effects identified with the march of the seasons.

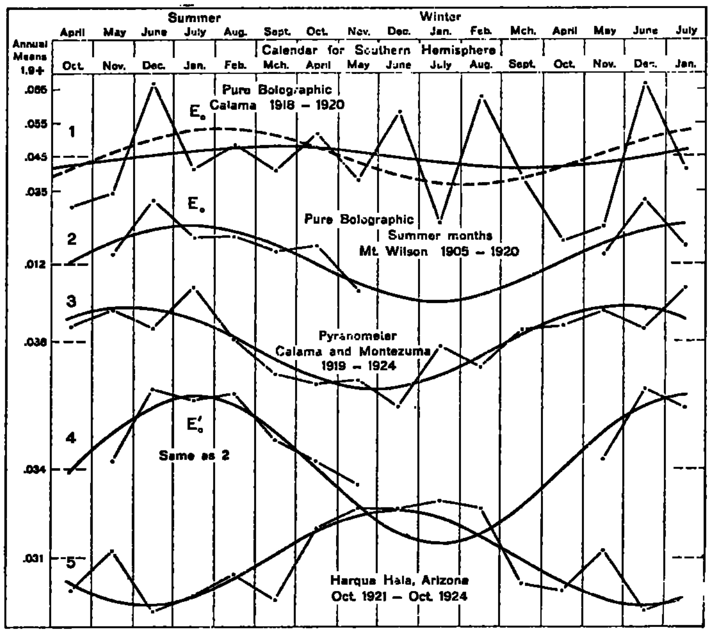


FIG. 7.—Monthly mean values of the solar constant in a sequence of 16 months in order to show fully the annual periodicity as a summer and winter terrestrial effect. The calendar for the Southern Hemisphere is shifted 6 months, to bring like seasons in both hemispheres into the same vertical lines. All scales same as at 1.

The observations at Mount Wilson from 1905 to 1920 were made only in the summer months, usually from June to October, but extended to include May in 1906, 1908, 1910 and 1912, and November during 1908, 1909, 1910, 1911, and 1913. The systematic seasonal change affecting the monthly means really stares the observant student in the face, and suggests at once an annual periodicity. Doubt on this point in the writer's mind was wholly removed in 1923 when the observations at Calama, Montezuma, and Harqua Hala from July, 1918 to September, 1922, were published.[11]

Since observations at Mount Wilson for the many missing months of the past years can never be supplied, no valid objection can be made to invoking least square methods to pass the best sine curve we can through the seven months' observations available. Curves 2 and 4 show the monthly mean values and the normal annual march for the two values E_0 from pure bolographic reductions and E'_0 a supposed superior value derived from E_0 by the application of a correction for atmospheric water vapor.

We regard even this one analysis of the meager data available in the case as a gratifying success; especially when it is so completely confirmed by the highly satisfactory fit of the sine curves to Calama, Montezuma, and Harqua Hala data, based on strong monthly mean values for every month of the year.

Katmai dust.—The doubtful values secured at Mount Wilson during the years 1912 and 1913 have been wholly omitted from the results shown in Figure 7. However, we have computed the sine curves, using all the data regardless of Katmai dust, with no very consequential difference of any kind. That is, the phases of the curves are hardly changed at all, and the amplitude of E_0 was increased from 0.0058 to 0.0069, and of E'_0 from 0.0108 0.0122. Doctor Abbot also objects[12] to my use of *all the Mount Wilson data* because it covers only one epoch of sunspot minimum, while there are two epochs of maximum. Therefore, we strike out all data for the year of strong sunspot maximum, 1917, and we still get the same period, with phases practically identical, amplitudes changed for E_0 from 0.0059 to 0.0055 and for E'_0 0.0109 to 0.0092. Thus the data refute the criticism.

It is difficult either to understand or to take seriously the following words by Doctor Abbot in answer to the evidences for the 12-month periodicity:[13] "I will not say there was absolutely nothing of the kind in Mount Wilson observations, but I regard it as nearly or quite nonexistent in later work. He [Marvin] has mistaken a real 11-month periodicity in recent years for a 12-month periodicity. Mr. Clayton discovered the 11-month periodicity over a year ago and reported it to me."

First, the period "is nearly or quite nonexistent. In the next sentence "it is a real 11-month periodicity." Is the period nonexistent, or is Marvin or Clayton mistaken about its length? It will take a lot of statistical evidence to prove "a real 11-month periodicity," whether evanescent or permanent, in solar constant values. When the length of the period has been proven to be, not 12 months, but 11, it will take another large mass of statistical evidence to prove that the periodicity is of solar rather than terrestrial origin. The case seems to stand in this way: Hardly denying that the Mount Wilson solar constant values show a 12-month periodicity, Doctor Abbot says that recent values show a real 11-month periodicity discovered by Clayton. Marvin insists that the real length of the period is 12 months, due to summer and winter atmospheric effects and submits the testimony of fully 3,000 daily values in proof and quantitative evaluation thereof.

Returning to the propriety of computing a 12-month periodicity from data for only 7 consecutive months, I want to say that we have no hope of securing an accurate evaluation of its constants. The existence of the period is the major present question. The results we give speak for themselves, truthfully representing all the data available. The features found are wholly in accord with like results for other stations making continuous observations over the entire year.

Incidentally, it must be emphasized that the fitting of a sine curve is the only correct method of getting the proper mean value of periodic data when values for a considerable part of the period are missing.

One of the noteworthy features about the Mount Wilson data (curves 2 and 4) is that the amplitude of the supposed inferior values of the solar constant E_0 is smaller than for later values (curves 3 and 5), and is but little over half the amplitude for the values of E'_0 which are supposed to be superior and are derived from the same parent data. The correction for water vapor applied to give E'_0 can hardly be considered as justified.

Curve 3 is noteworthy because it is based on pyranometer observations only, designated W. M. in the Smithsonian publications. Observations averaged 18 daily values per month for an unbroken period of 60 months, a total of 1,080 daily values. The fit of the sine curve must be regarded as highly satisfactory.

[11] MONTHLY WEATHER REVIEW, February, p. 71, and April, 1923, p. 188; Annual Period."

[12] Abbot, C. G., Solar variation and forecasting, Smithsonian Miscellaneous Collections, vol. 77, No. 5, pp. 11–12.

[13] Solar variation and forecasting, loc. cit., p. 9.

The foregoing remarks apply also to curve 5 based on 4 years pyranometer observations at Harqua Hala, with a total of 764 daily values.

What is the striking lesson the diagram as a whole teaches? Very clearly it is, that almost without exception monthly mean values of the solar constant exhibit a very definite annual period, unfailingly associated with summer and winter states of the atmosphere. The reader must remember that when stations in the Northern and Southern Hemispheres are being compared, there is an absolute time interval of six months between the values in any vertical line or band of the diagram. The sun, therefore, can have no part in causing the almost perfect synchronism which the eye catches at once, including the equally striking exception apparent in curve 5. All these effects are due to the one state common to all observations wherever and whenever made, namely, summer and winter atmospheric conditions: high solar constants with summer conditions, low constants with winter conditions. The clash and inconsistency in the trend of the monthly means as observed at Harqua Hala, as compared with the trend at its sister station, Mount Wilson, only 250 miles to the west, is complete and physically irrational. There is no lack of annual period in the monthly means at Harqua Hala. The smooth sine curve fits the observations with entire satisfaction. The summer and winter effect is all there. The inconsistency lies in the fact that it is the summer and winter atmospheric states at Montezuma in the Southern Hemisphere that influence the summer and winter values of the solar constant at Harqua Hala in the Northern Hemisphere. The explanation of this anomalous result is found in the following quotation from the annual report of the Astrophysical Observatory for 1924 (p. 105):

> As soon as we began to receive daily telegrams from both stations occasional fairly wide disagreements of individual days commanded attention. We felt it necessary, in studying the causes of such disagreements, to revise again entirely the systems of little corrections to solar-constant values which have to be made to allow for the haziness and humidity of our atmosphere. This revision could be made with more advantage because much additional data had meanwhile accumulated. * * *
>
> A new method of determining these corrections has been devised, which eliminates satisfactorily the influence of the solar changes which have occurred. Hitherto this matter of solar change superposed upon the small terrestrial sources of error which we desire to eliminate has been very embarrassing. Of course, if one could wait many years before proceeding to evaluate the terrestrial effects, the solar changes, being independent or but loosely connected with local terrestrial ones, would be eliminated in the mean of a mass of observations. We can, indeed, after several years more of observing, finally proceed in this way. But wishing to make immediate use of our results a new method of procedure has fortunately occurred to us which permits us to avoid the interference of solar changes altogether. The details will be published soon.

The method of making these corrections is described in the pamphlet on Solar Variations and Forecasting (p. 14). The method may seem to have been entirely valid in principle at the start, but the sequel of its actual application proves that it is clearly erroneous in its effects and we are reluctantly forced to take the position that *the provisional values of the solar constant for Harqua Hala as published in volume 77, No. 3, Smithsonian Miscellaneous Collections, can not be accepted as independent of those for Montezuma, and that the two values are correlated in an entirely artificial way.*

It seems to the writer futile to try to contest the overwhelming evidence in support of a 12-month period in the published values of the solar constant. Our analysis uses all the data available. The periodicity is present in the old as well as in the latest values. The amplitudes and phases are entirely comparable and consistent and can not be altered to any consequential extent by any valid selection or rejection of particular data. Only extraordinary reasons justify selection or rejection of data in a case of this kind; the data for the Katmai years were excluded from Table 5 and Figure 7 chiefly to show how inconsequential that great disturbing factor really was. Its influence was small because we are dealing with an inherent, fundamental characteristic *affecting all the daily values all the time.*

All methods of extrapolation to zero air mass fail.—Every method yet devised or employed for computing solar constants unfailingly extrapolates to zero air mass highly important and significant atmospheric states and surface terrestrial conditions. The artificial variations thus imposed upon values of the solar constant should not be offered and can not be accepted as evidence for real day to day solar fluctuations.

It seems very plain from this investigation that the pure bolographic reduction is most free from systematic faults and errors in day to day values although its variations due to errors are larger than for the pyranometer. The latter instrument, as previously mentioned, is simply an empirical substitute for the Langley-Bouguer straight line law of extrapolation to zero air mass. Our studies show that the 12-month period is freely extrapolated by this instrument, and other studies not presented here in detail show that nearly all groups of pyranometer values plot in decidedly skew forms of frequency distribution. This skewness is conspicuously a characteristic of *all surface readings of intensities*, especially at the higher air masses. It is clearly evident in the parent data (fig. 2), even for air mass only 1.5. The origin of this skewness is terrestrial, and therefore should not be extrapolated to zero air mass.

It is easy to see what would have happened had the telegraphic Harqua Hala observations been *rigorously independent* of those coming in from Montezuma. Assuming equality in other respects, daily telegraphic values would have seemed to agree nicely in the spring and autumn seasons, only to show wide systematic discrepancies during the summer and winter seasons. Obviously, the mean of independent daily values from the two stations, despite wide occasional differences, would tend to be nearly or entirely free from *an annual period.* I do not mean to imply that the actual original observations would show the above results, because it is not at all likely that the two stations and equipments are alike in other respects. It is quite certain that not only the equipment but the atmospheres at the two stations exert quite different effects upon daily values.

In the face of all the evidence we have presented to show the real nature of fluctuations in solar constant values it is a serious error of interpretation of original observations to insist that any of the published values of the solar constant fairly represent day-to-day changes in solar intensity.

The only course the writer can advocate is to see if it is not possible, as seems to be the case, to so modify the present methods of extrapolation to zero air mass as to mostly remove the existing serious objections. This of course is possible only to those having free access to original and unpublished observational data. Several promising possibilities seem to be opened up in the examples of analysis we have given.

VI. SOLAR VARIATIONS COMPUTED FROM OBSERVATIONS AT INDEPENDENT STATIONS

Disregarding a small constant difference between mean solar constants from groups of same-day observations at Montezuma and Harqua Hala, also the artificial correlation previously mentioned, we give in Table 7 the results of the application of equations (10), (11), and (12) to the evaluation of the station errors σ_x Montezuma, σ_y Harqua Hala and σ_i possible solar variability.

For comparative purposes the 399 observations made on the same days at both stations are divided into three groups representing more or less homogeneous values. A limited number of simultaneous bolographic observations were made at Mount Wilson and Calama during the years 1918, 1919, and 1920. By combining all the values into one group, being careful to preserve the lowest possible minimum sum of squares of variations and differences, by excluding effects due to large secular changes between years and to scale differences between stations, we get the results given in the bottom line of values in Table 7 indicating a possible solar variation of 0.55 per cent, which is from two to four times the variation shown by the other data. Comparing these results with the magnitude of the station errors we see that σ_i is a function of those errors, a fact which, as pointed out in Section I, invalidates the assumption that equations (10), (11), and (12) are three simultaneous equations between three independent unknowns. The results in Table 7, therefore, must be interpreted to mean that either solar variation is nonexistent or is relatively so small that it can not be disentangled from the irregular larger variations in daily values due to errors of observation.

TABLE 7.—*Calculation of solar variations from synchronous observations at Harqua Hala or Mount Wilson, designated by subscript x and at Calama or Montezuma, designated by subscript y.*

Date	Number of observations	Solar constant E_o		Total variation T			Calculated values			$\frac{0.6745\sigma_i}{E_o}$
		$\overline{E}_x$	$\overline{E}_y$	T_x	T_y	T_{xy}	σ_x	σ_y	σ_i	
										Per cent
Oct. 4, 1920, to Mar. 31, 1922	100	1.9460	1.9467	0.0129	0.0171	0.0176	0.0096	0.0148	0.0086	0.30
Apr. 1, 1922, to July 1, 1923	106	1.9168	1.9210	.0115	.0143	.0173	.0107	.0136	.0043	.15
Aug. 1, 1923, to Nov. 30, 1924	193	1.9251	1.9231	.0108	.0139	.0156	.0091	.0126	.0058	.20
July 27, 1918, to Sept. 6, 1920	66	1.9457	1.9417	.0301	.0241	.0314	.0256	.0174	.0158	.55

VII. CONCLUSION

Final definitive evidence, especially in quantitative measures, can not be secured from observations at a single station with only one set of observing instruments. Much could be learned from check observations by wholly independent equipments maintained side by side, and it is hoped such check determinations can be secured at some station to be established in the future.

A considerable number of synchronous observations have been secured from stations in pairs, as Bassour and Mount Wilson, the latter and Calama, including Calama and Montezuma with Harqua Hala. Unfortunately, because of volcanic dust and other untoward circumstances, these synchronous values are so much affected by important accidental and systematic terrestrial and artificial causes as to more or less invalidate the evidence which these observations might show of a small possible variation, which can be entertained as real only when confirmed by future *independent* observations at other stations.

It is indicated in the text that pyrheliometer readings alone are nearly errorless values from which real solar variability can be proven and evaluated with considerable accuracy, especially if observations are secured from uniformly standardized instruments observed at several wholly independent stations in the arid regions of the earth and over as great a range of elevation as possible.

The International Commission for Solar Radiation has this subject under consideration, and the writer hopes its actions may lead to progress in this important field of geophysical science.

The presentation in this paper is offered as an example, so to speak—a preliminary survey and study. I expect to extend the investigation to many other observations thus far examined not at all or only in the most superficial way.

SMITHSONIAN SOLAR-CONSTANT VALUES

By HERBERT H. KIMBALL

[Washington, Sept. 1, 1925]

SYNOPSIS

This paper considers briefly the magnitude of errors in solar constant determinations arising from errors in the fundamental pyrheliometric readings and in their extrapolation to zero atmosphere.

The degree of correlation between solar constant determinations made nearly simultaneously at Montezuma, Chile, and Harqua Hala, Ariz., leads to the conclusion that only an insignificant part of their day-to-day variations can be attributed to some such common cause as solar variability.

INTRODUCTION

A critical study of the work during the past 20 years of Doctor Abbot and his associates in connection with determinations of the value of the solar constant leads one to a profound respect for the skill, energy, and devotion to science that is evident throughout. It is not a simple matter to obtain from measurements of solar radiation intensity made at the bottom of the sea of air the intensity of that radiation before it enters our atmosphere. This is what they have done, however, and with such precision that the published mean value of their determinations, 1.94 gram-calories per minute per square centimeter, is almost universally accepted, although this value is necessarily subject to a probable error that as yet can hardly be evaluated. That Doctor Abbot and his associates seem to recognize this is indicated by their statement that after all possible care in the standardization and intercomparison of instruments employed at Montezuma and Harqua Hala it was necessary to add a little more than 1 per cent to the solar constant determinations made at the latter station to bring them into accord with those at the former station.[1]

[1] Abbot, C. G., and Colleagues. Values of the solar constant, 1920-1922. MONTHLY WEATHER REVIEW February 1923, 51: 71-81. (See especially p. 74.)